Measurement Good Practice Guide No. 71

The Measurement of Mass and Weight

Stuart Davidson, Michael Perkin
Division of Engineering and Process Control
National Physical Laboratory

Mike Buckley
South Yorkshire Trading Standards Unit

Abstract: This Good Practice Guide is intended as a useful reference for those involved in the practical measurement of mass and weight.

ISSN 1368-6550

June 2004

National Physical Laboratory
Teddington, Middlesex, United Kingdom, TW11 0LW

Website: www.npl.co.uk

Acknowledgements

The authors acknowledge the financial support of the National Measurement System Directorate of the UK Department of Trade and Industry.

The Measurement of Mass and Weight

Contents

1 Introduction

The unit of mass, the kilogram, remains the only base unit in the International System of Units (SI), which is still defined in terms of a physical artefact. Its definition is:

"The kilogram (kg) is the unit of mass; it is equal to the mass of the international prototype of the kilogram."

The difference between 'mass' and 'weight' is that mass is a measure of the amount of material in an object, weight is the gravitational force acting on a body. However, for trading purposes weight is often taken to mean the same as mass.

The international prototype of the kilogram is kept at BIPM, the International Bureau of Weights and Measures in Sèvres, Paris. It consists of an alloy of 90% platinum and 10% iridium in the form of a cylinder, 39 mm high and 39 mm in diameter. It is stored at atmospheric pressure in a specially designed triple bell-jar.

About 60 countries hold platinum-iridium alloy copies of the BIPM kilogram (K), whose values have been determined directly from K. The National Physical Laboratory (NPL) holds the UK copy (No. 18), which is referred to as the national prototype kilogram, or simply kilogram 18 and is the basis of the entire mass scale in the UK. The NPL participates in a wide range of international comparisons to ensure that measurements made in the UK are equivalent to those made elsewhere in the world. In the past there have been some problems with organisations based in one country not accepting traceability to any NMI other than their own. This situation has been addressed with the advent of a structured approach to international equivalence via a Mutual Recognition Agreement (MRA) and regular international measurement comparisons.

Figure 1: The UK national standard kilogram

The use of appropriate mass standards and their correct treatment at all times is essential to mass metrology. Weights are divided into classes from high quality reference standards (Class E_1) to those used in industrial settings (Class M_3). These classes were originally specified for legal metrology purposes, but they are now in common usage throughout mass measurement. The International Organisation for Legal Metrology (OIML) document OIML R111 [1] specifies the properties of weights of each class and specifies the tests that are necessary prior to carrying out a mass calibration.

2 Weights

2.1 Material

Weights should be made of a material that is chemically inert, non-magnetic, hard enough to resist scratching and of a density that meets the OIML R111 recommendations for its class. Austenitic stainless steel is generally used in the construction of Class E_1 and E_2 weights. Lower accuracy weights may be manufactured from plated brass, iron or other suitable materials. Unplated brass should be avoided due to its susceptibility to atmospheric induced surface instability.

Figure 2: A stainless steel secondary standard weight set

Class E_1 and E_2 weights must be integral in construction, i.e. be made of a single piece of material. Other weights classes can be made up of multiple pieces with a sealed cavity to allow for adjustment. As with the other properties of weights the shape of weights for particular classes is defined in OIML Recommendation R111.

2.2 Surface condition

Prior to use weights should be inspected for surface damage such as scratching or contamination. It is almost inevitable that weight surfaces will become slightly scratched in use, but weights with gross scratching can potentially become unstable due to contamination filling the scratch mark.

2.3 Magnetism

There are two magnetic properties that must be measured to characterise weights, permanent magnetisation and magnetic susceptibility. Magnetic susceptibility is a measure of whether the weight can become magnetised by being placed in a magnetic field (magnetism of this sort is transient) while permanent magnetism is a feature of a weight that cannot be altered. It may be possible to de-Gauss magnetically susceptible weights using a commercial de-Gausser, but such treatment has no effect on permanent magnetisation. OIML Recommendation R111 sets out permissible limits for these two properties for various classes of weights.

2.4 Weight cleaning

Stainless steel weights (OIML E and F Classes) should routinely be dusted before use using a clean soft haired brush. With the exception of this, the cleaning of weights should be avoided unless it is absolutely unavoidable as it will affect their calibration history. Where cleaning is unavoidable, it is recommended that weights should be calibrated both before and after cleaning. Rubbing with a soft clean cloth is often enough to remove marks. Solvent cleaning should be used as a last resort. In general this will take the form of wiping the weight with a clean cloth that has been soaked in solvent.

After cleaning it is necessary to allow weights to stabilise before calibrating them. The stabilisation time will vary, from a few hours to several days, according to the class of weight and the extent of cleaning that has been undertaken [6].

Cast iron weights, if in good condition, may be cleaned by brushing with a stiff brush. Rust can be removed with a wire brush.

2.5 Weight handling

Weights must always be handled with the greatest care. It is important that they are never:

a) touched with bare hands
b) handled with sharp or abrasive tools and materials
c) in contact with tools or surfaces that are not scrupulously clean
d) slid across surfaces
e) knocked together
f) breathed on or spoken over by the operator
g) cleaned by inappropriate means

The following handling methods are recommended in order to avoid the problems mentioned above:

2.5.1 Gloves

Gloves should be worn whenever practicable. Chamois leather is an ideal material for such gloves as it has good thermal insulation properties (so reducing thermal influences on the balance during the weighing process) and affords a good grip on large weights when manipulating them.

2.5.2 Tweezers

Tweezers, forceps or other specialist lifting devices should be used whenever practicable to pick up weights and manipulate them inside the balance. It is important that these tools have no sharp edges and should, when possible, not have metal to metal contact with the weights.

Ungloved hand, potential heat transfer to weight and enclosure.

Users hand in balance enclosure affecting the thermal stability.

Gloved hand outside enclosure **Recommended practice.**

Figure 3: Good practice in weight handling

2.6 Clean surfaces

Surfaces in a mass laboratory should be kept clean and dust free at all times. However, it is advisable to place acid free tissue paper, or something similar, on any surface prior to putting weights on it.

2.7 Weight storage

Weights should be stored in a manner that protects them from damage or contamination and allows them to be easily identified. Manufacturers generally supply weights in felt lined wooden boxes. Such boxes normally have a recess for each weight and a lid which touches the weights when closed to hold then in place. Smaller weights tend to be loosely confined within a small aperture in their box.

The major disadvantage with using boxes for storage is that they contact virtually the entire surface of the weight so potentially causing contamination problems. It is important that the interiors of weight boxes are kept clean to prevent a build up of dust and other particles.

It is preferable for weights to be stored on acid free tissue paper under glass domes. This is an ideal situation as only the base of the weight is in contact with other surfaces. Regardless of how the weight is stored it is important to ensure that it is clearly identified at all times so that it cannot get confused with similar weights.

3 Weighing techniques

There are many weighing techniques currently employed in mass metrology. This is indicative of the wide range of processes that rely on weighing and the uncertainty demands that are required by different industrial sectors and end-users.

3.1 Direct reading measurements

The most simple method of weighing is to simply place a test piece on a mass balance and take the displayed reading as its weight. This type of measurement is only suitable for low accuracy applications but even in this most straight-forward application it is essential to follow good practice.

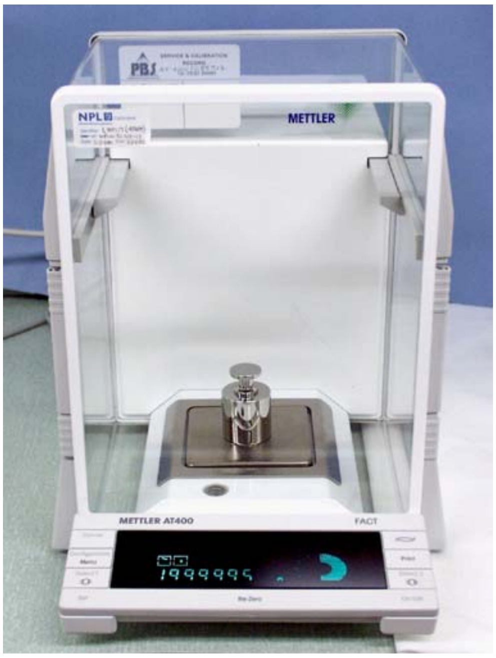


Figure 4: Electronic balance

As with any form of mass measurement it is essential to have the balance calibrated on a regular basis. It is recommended that the balance undergoes a full assessment and calibration, by a suitably accredited body, on a periodic basis which will be influenced by the application and frequency of use. It is important that the balance scale is tared before use. If there is an internal balance calibration feature this should be used prior to making measurements on the balance. A reading with zero load should be taken (z_1) followed by the reading with the load on the pan (r_1) and a final zero reading (z_2). This allows the user to compensate for any drift in the instrument. The drift corrected reading (r_d) is given by:

$$r_d = r_1 - \frac{z_1 + z_2}{2}$$

3.2 Weighing by differences

This technique is particularly common in analytical chemistry. In general it involves placing a container on the balance pan, noting the reading, and then adding a substance to it. The final balance reading is noted and the difference between the two readings is taken to be the amount of material in the container. This approach is fine for relatively low accuracy requirements but is not ideal for more demanding measurements. This type of measurement is susceptible to problems with temporal balance drift (which is generally more of a problem with modern electronic analytical balances than was the case with mechanical balances).

A more robust method of carrying out measurements of this type is to have two similar containers, one to fill with the test substance and one to act as a reference. The weighing should then be carried out as follows:

a) The reference container is placed on the balance pan and the reading noted (ref_1).
b) The empty container (to be loaded with the test substance) is placed on the balance ($test_1$)
c) The container is filled with the test substance and the balance reading taken ($test_2$)
d) The reference container is put back on the pan and the reading taken (ref_2).

The weight of substance added to the container (w_t) may then be calculated as follows:

$$w_t = (test_2 - test_1) - (ref_2 - ref_1)$$

This weighing scheme eliminates drift and should allow better weighing uncertainties to be achieved.

3.3 Substitution weighing

High accuracy weighing is generally carried out by comparing the test-weight with mass standards of similar nominal value. This is known as substitution weighing.

When weight A (the first weight to be place on the balance pan) is compared with weight B (the second weight used) there will generally be a difference (Δm) between the readings with each of the weights on the pan. This may be expressed in terms of the following equation:

$$A = B + \Delta m$$

It is important to follow a consistent nomenclature when carrying out weighings and expressing the comparison in terms of a simple equation is perhaps the best way to do this. Throughout this section the concept of an equation is used to describe a comparison weighing.

If the test-weight is of an unusual nominal value it will be necessary to use a standard made up of several calibrated weights, or to use additional weights to make the test-weight up to a standard's nominal mass. In these cases it is recommended that a direct reading be taken as a coarse check to ensure all the weights have been noted down.

The number of comparisons carried out in order to calibrate a weight, and the number of independent mass standards used will depend on the uncertainty requirements of a particular calibration.

There are two popular techniques for comparative calibration: ABA… and ABBA calibrations.

3.4 ABA… calibration

This calibration involves the comparison of two weights, normally an unknown and a standard, by placing each one in turn on the balance pan and noting the reading. The process is symmetrical in that the weight that is placed on the pan first is also on the pan for the final reading. It is possible to vary the number of applications of each weight according to the application, but three applications of the test weight and two of the standard would be a typical regime. This leads to the following measurement scheme:

$$A_1\ B_1\ A_2\ B_2\ A_3$$

where A_i and B_i represent the balance reading for the i^{th} application of weights A and B respectively

The difference between weights A and B is then calculated from the equation:

$$\Delta m = \frac{A_1 + A_2 + A_3}{3} - \frac{B_1 + B_2}{2}$$

This may be generalised for an ABA comparison involving n applications of weight B to become:

$$\Delta m = \frac{\sum_{i=1}^{n+1} A_i}{n+1} - \frac{\sum_{i=1}^{n} B_i}{n}$$

3.5 ABBA calibration

The ABBA calibration method is potentially more efficient than the ABA method in terms of the number of weight applications required to produce a calibration result of a particular uncertainty. In this case the weights are applied to the balance pan in the following order:

$$A_1\ B_1\ B_2\ A_2$$

Where: A_n and B_n represent the balance reading for the n^{th} application of weights A and B respectively.

It is important in this weighing method that weighings B_1 and B_2 must be independent of each other. To achieve this on a typical electronic comparator the weight must be removed from the pan between readings. If a balance with an arrestment mechanism is being used it is usually sufficient to arrest the balance and open and close the balance case between the measurements. The mass difference (Δm) is calculated from the following equation:

$$\Delta m = \frac{A_1 + A_2}{2} - \frac{B_1 + B_2}{2}$$

3.6 Cyclic weighing

High accuracy (OIML Class E and F) mass calibration usually involves calibration against two or more mass standards. Often it is necessary to calibrate several weights of the same nominal value. In this case it is inefficient to calibrate each weight against two standards. A technique known as cyclic weighing is used to rationalise the number of measurements necessary.

Comparison	Measured Difference	Corrected Difference	Using S1 value	Using S2 value	Measured value
S1 v T1	d_1	d_1 - C	T1= S1-d_1+C	T1= S1*-d_1+C	
T1 v T2	d_2	d_2 - C	T2=T1-d_2+C	T2=T1-d_2+C	The average
T2 v T3	d_3	d_3 - C	T3=T2-d_3+C	T3=T2-d_3+C	of the values
T3 v S2	d_4	d_4 - C	S2*=T3-d_4+C	S2=T3-d_4+C	for T1 to T6
S2 v T4	d_5	d_5 - C	T4=S2*-d_5+C	T4=S2-d_5+C	calculated in
T4 v T5	d_6	d_6 - C	T5=T4-d_6+C	T5=T4-d_6+C	the previous
T5 v T6	d_7	d_6 - C	T6=T5-d_7+C	T6=T5-d_7+C	columns
T6 v S1	d_8	d_8 - C	S1=T6-d_8+C	S1*=T6-d_8+C	
Sum	d_1+ d_2+...+ d_8	0			
Correction	C = sum/8				

* The value for S2 calculated using S1 (and vice versa) is checked against the true value of S2

The table shows a typical weighing scheme for a cyclic weighing. Although two standards are used the number of weighings required is reduced from 12 to 8 through the use of this scheme. The weighing scheme is symmetrical and, as each weight is used the same number of times, the sum of the measured weight differences should be zero. In practice this is not usually the case and a correction, C, is calculated to allow for temporal changes. The weighings must be carried out in one series to allow this correction to be made. The values for the test weights T can be calculated in turn from each of the two standards and an average taken. The scheme contains two checks on the goodness of the weighings, the size of the correction C and the agreement between the values of T calculated from the two standards.

3.7 Weighing by sub-division

Sub-division is used for the most demanding applications. It involves the use of standards of one or more values to assign values to weights across a wide range of mass values. A typical example of this would be to use two or three 1 kg standards to calibrate a 20 kg to 1 mg weight set. Equally it would be possible to use a 1 kg and a 100 g standard for such a calibration. Typically this is used for OIML E Class calibrations.

This is most easily illustrated by considering how values would be assigned to a weight set using a single standard. In reality the weighing scheme would be extended to involve at least two standards. The standard is compared with any weights from the set of the same nominal value and also with various combinations of weights from the set that sum to the same nominal value. A check-weight, which is a standard treated in the same manner as any of the test-weights, is added in each decade of the calibration so that it is possible to verify the values assigned to the weight set. In the case of a 1 kg to 100 g weight set the following minimal weighing scheme may be used:

1000 =1000S
1000 = 500 + 200 + 200D +100C
1000S = 500 + 200 + 200D +100
500 = 200+ 200D+100
500 = 200+ 200D+100C
200 = 200D
200 = 100 +100C
200D = 100 +100C
100 = 100C

In a simple case such as this it is possible to calculate the values of the test weights manually, but in a realistic situation when several standards are used and information is required about the weighing scheme uncertainty it is necessary to undertake a least squares analysis using a computer.

The sub-division weighing scheme has the following advantages:

a) it minimises use on (and hence wear on) standards
b) it produces a set of data which provides important statistical information about the measurements and the day to day performance of the individual balances
c) there is a redundancy of data (ie more measurements than unknowns).

It has the following disadvantages:

a) it requires a relatively complex algorithm to analyse the data

b) it necessitates placing groups of weights on balance pans (this can cause problems for instruments with poor eccentricity characteristics or automatic comparators designed to compare single weights.

3.8 Periodicity of Calibration

The period between the calibration depends on the amount and type of use the weights experience and the accuracy class of the weights. As a general rule new weights should be calibrated annually until a reasonable calibration history (at least three calibrations) has been build up. After this, depending on the stability of the weights, the calibration period can be extended to two or even four years.

3.9 Scale errors and the use of make-weights

Make-weights are weights that are added to a load in order to make it approximately equal in weight to the object it is being compared with. Make-weights are added so that only a small part of the comparator's scale is used during a comparison. For example if a 100 g scale had a 1 % error it would equate to a 0.2 g error if there were a 20 g difference between the loads under test, but this could be reduced to a 0.01 g error if a make-weight is used to balance the loads to within 1 g.

If a make-weight of value M_w is added to load A during a comparison the weighing equation becomes:

$$A + M_w = B + \Delta m$$

so in order to calculate the value of A from B it is necessary to subtract the value of any make-weights used in association with A from the sum of the value of B and the calculated mass difference. Similarly, if make-weights had been used with weight B the value of the make-weights would be subtracted from the value of A before taking the difference in the balance readings into account.

4 Air density measurement and buoyancy correction

The measurement of air density is necessary in the field of mass measurement to allow buoyancy corrections to be made when comparing weights of different volume in air. It is particularly important when comparing weights of different materials or when making mass measurements to the highest accuracy.

4.1 True mass

The mass of a body relates to the amount of material of which it consists. In terms of the calibration of weight it is referred to as true mass in order to differentiate it from conventional mass, which is generally used to specify the value of weights (see below). The international prototype of the kilogram for which the mass scale throughout the world is realised is defined as a true mass of exactly 1 kilogram. All high accuracy comparisons should be performed on a true mass basis (including class E1 and E2 calibrations) although values may be converted to conventional mass when quoted on a certificate.

4.2 Conventional mass

This is the value normally quoted on a certificate and is the conventional value of the result of a weighing in air, in accordance with International Recommendation OIML R 33. For a weight at 20 °C, the conventional mass is the mass of a reference weight of a density of 8000 kg/m^3, which it balances in air of a density 1.2 kg/m^3.

A conventional mass value for an artefact can be calculated from a true mass value using the following equation:

$$M_c = M_t \left(1 + \left(\frac{1}{8000} - \frac{1}{\rho} \right) 1.2 \right)$$

where: M_c is the conventional mass of the artefact in grams
M_t is the true mass of the artefact in grams
ρ is the density of the artefact in kg/m^3

4.3 Buoyancy correction

This is a correction made when comparing the mass of artefacts of different volumes. It is equal to the difference in the volumes of the artefacts multiplied by the density of the medium in which they are compared (usually air). When comparing true mass values the buoyancy correction to be applied between two artefacts can be given by the following equation:

$$BC = (V_1 - V_2) \times \rho_{air}$$

Where: BC is the buoyancy correction to be applied
V_n is the volume of the artefact n (ie $M_\mathrm{n} / \rho_\mathrm{n}$)
ρ_{air} is the density of the air at the time of comparison

The calculated buoyancy correction should be applied in the form:

$$M_{t_1} = M_{t_2} + BC$$

Where: $M_{t\mathrm{n}}$ is the true mass of artefact n

When calibrating standard weights comparisons are normally performed on a conventional mass basis. For such comparisons the buoyancy correction depends on the difference in air density from the conventional value of 1.2 kg/m^3. Because of the way the conventional mass is specified, comparisons made in air of density exactly 1.2 kg/m^3 require no buoyancy correction no matter what the volume difference of the weights being compared. For comparisons performed on a conventional mass basis the buoyancy correction is given by the following equation:

$$BC = (V_1 - V_2) \times (\rho_{air} - 1.2 \times 10^{-3})$$

The buoyancy correction is applied with the same convention as for the true mass correction, ie:

$$M_{c1} = M_{c2} + BC$$

The equation for conventional mass buoyancy correction is more complicated than for true mass and care must be taken with the sign of the correction. If weight 1 has a greater volume than weight 2 and is compared in air of density greater than 1.2 kg/m^3 the correction (*BC*) will be positive.

4.4 The application of buoyancy corrections

The table below shows the magnitude of the buoyancy correction when comparing weights of stainless steel with those of another material in air of standard density (1.2 kg/m^3) on a true mass basis.

Material compared with Stainless steel	Buoyancy correction (ppm)
Platinum Iridium	94
Tungsten	88
Brass	8
Stainless Steel	7.5*
Cast Iron	24
Aluminium	294
Silicon	365
Water	875

*This is the result of comparing two types of stainless steel, with densities 7.8 and 8.2 g/cm^3

Table 1: Buoyancy Corrections when Comparing Dissimilar Materials in Air

Figure 5: stainless steel and platinum kilograms

The table shows that even when comparing weights of nominally the same material (such as stainless steel) attention must be paid to buoyancy effects when the best uncertainty is required.

When comparing weights of dissimilar materials the effect of air buoyancy becomes more significant and must be applied even for routine calibrations when true mass values are being measured.

When working on a conventional mass basis the buoyancy corrections become smaller. International Recommendation OIML R 33 use a range for air density of 1.1 to 1.3 kg/m^3 (ie. approximately ± 10% of standard air density) meaning the corrections are about one tenth of the true mass corrections. This together with the limits specified by OIML R 33 for the density of weights of Classes E1 to M3 mean that the maximum correction for any weight is one quarter of its tolerance. This is generally not significant for weights of Class F1 and below (although allowance should be made for the uncertainty contribution of the un-applied correction) but for Class E1 and E2 weights buoyancy corrections need to be applied to achieve the uncertainty values required for weights of these Classes.

4.5 Determination of air density from parametric measurements

The standard method for determining air density involves the measurement of temperature, pressure and humidity. From these measurements, and taking into account carbon dioxide concentration for the best accuracy, the density of the air can be calculated. The empirical formula for the calculation of air density recommended by the *Comité International des Poids et Mesures* (CIPM) was derived by Giacomo [2] and modified by Davis [3]. Table 2 shows typical and best achievable uncertainties for the calculation of air density from the above parameters using the CIPM formula.

	Routine Measurement		Best Capability	
	Uncertainty	ppm	Uncertainty	ppm
Temperature (^oC)	0.1	360	0.01	36
Pressure (mbar)	0.5	500	0.05	50
Humidity (% rh/^{o}C dew pt.)	5%	350	0.25^{o}C	58
CO_2 content (ppm)	-	-	50	21
CIPM Equation		100		100
Total (x 10^{-3} kg/m^3)	**0.86**	**720**	**0.16**	**133**

Table 2: Routine and Best Achievable Realisation of Air Density using the CIPM Formula

5 Balance assessment

The need for regular and appropriate assessment of balances is vital both in providing traceability of results and in providing accurate and reliable results. Balances can be used either as comparators or as direct reading devices. The assessment of the balance will depend on the mode in which it is used. When used as a comparator traceability for the calibration will be provided by the mass standard against which the unknown is compared and the performance of the balance is effectively given by its repeatability. When used as a direct reading device traceability for the measurement is in effect provided by the balance itself and a number of other factors such as the linearity, stability and temperature sensitivity of the balance become important. In all cases it is vital that:

- the balance assessment reflects the way in which the balance is used in practice
- it takes place in the same conditions as the balance would normally be used

It is essential that both these conditions are met if the results of the balance assessment are to give an accurate reflection of the performance of the balance and therefore provide a valid contribution to an uncertainty budget for measurements performed on the balance.

5.1 Balance location

To perform at its best the balance should be located in a position as free from vibration, thermal instability, direct sunlight and draughts as possible. Care should also be taken to avoid external influences such as magnetic fields and radio frequency interference. The performance of the balance depends to a great extent on the environment in which it is located so some thought should be given to optimising its position with respect to the above parameters while allowing practical use of the balance. As previously stated the balance should be assessed in the location in which it will be used. If a balance is used in a shop floor environment it should not be assessed in a temperature controlled laboratory environment.

5.2 Measurement equipment

A basic assessment of the performance of a balance for repeatability, hysteresis and eccentric loading can be performed with two un-calibrated weights of similar nominal values. A more comprehensive assessment, however, requires a set of calibrated weights and a temperature probe to monitor the temperature of the balance. Since the sensing elements of most balances are temperature sensitive it is vital to keep a record of the balance temperature while it is being assessed as unusual variations in temperature may result in poor performance. A calibrated set of weights allow the scale error of the balance to be assessed.

5.3 Two-pan balances

Figure 6: Two-pan mechanical balance

These balances consist of a symmetrical beam and three knife edges. The two terminal knives support the pans and a central knife edge acts as a pivot about which the beam swings. Two-pan balances are generally un-damped with a "rest point" being calculated from a series of "turning points". Some balances incorporate a damping mechanism (usually mechanical or magnetic) to allow the direct reading of a "rest point".

Readings from this type of balance tend to be made using a simple pointer and scale although some use more complicated optical displays. In all cases the reading in terms of scale units needs to be converted into a measured mass difference.

Due to the construction of the balance they are always used as comparators (comparing the load on one pan with the other) but the way in which the comparison is achieved can vary - double-double, double, double-substitution and substitution modes can all be used. It is vital that the balance is assessed using the same mode of comparison as is used for the normal operation of the balance.

5.4 Single-pan mechanical balances

Figure 7: Single-pan mechanical balance

These generally consist of a beam with two knife edges, one to support the weighing pan and one acting as a pivot. A fixed counterweight balances the load on the pan.

This type of balance is usually damped and a series of built in weights allow the calibration of a range of mass values by maintaining a constant load on the balance beam.

The displays on these balances tend to be of the optical variety and the sensitivity of the balance can usually be adjusted by the user.

5.5 Single-pan electronic balances

These are usually top loading balances with the applied load being measured by an electro-magnetic force compensation unit or a strain gauged load cell. Single pan electronic balances give a direct reading of the weight applied whereas the other two mechanical balance types rely on the comparison of two forces (an unknown weight with either an external or internal weight). Despite the possibility of using these balances as direct reading devices (applying an unknown weight and taking the balance reading as a measure of its mass) single pan electronic balances will always perform better when used as comparators, comparing a standard and an unknown in an ABA or ABBA sequence. As with all the balance types discussed the method of assessment should reflect the way in which the balance is used in practice.

5.6 Assessment of two pan balances

This type of balance is less frequently used mainly due to the amount of time it takes to make a weighing compared with electronic balances. For this reason only a brief outline of the tests that should be performed on this type of balance is given. Further details of these tests may be found in NPL Notes on Applied Science No.7, 'Balances, Weights and Precise Laboratory Weighing', 1954.

5.6.1 Routine periodic checks

The following tests should be carried out on a regular basis and are essential to the routine operation of the balance.

5.6.2 Rider weight

The effective value (mass value corrected for air buoyancy based on the actual density of the weight) of the rider weight should be measured against suitable mass standards.

5.6.3 Internal balance weights

Instead of (occasionally as well as) a rider bar some two-pan balances have built in poising weights. These should be removed and calibrated in the same way as the rider weight.

5.6.4 Linearity of scale

This can be checked using either fractional weights or with the rider and rider bar.

5.6.5 Sensitivity

Measuring the sensitivity of the balance allows a measured difference in terms of scale divisions to be converted into a mass difference and is therefore vital to the use of the balance. A suitable sensitivity weight, which can be easily transferred between the balance pans without arresting the balance, is required. The value of this weight should be enough to cause the pointer to move along the scale by between one quarter and one third of the total scale length. The sensitivity weight should be calibrated against suitable mass standards. The weight is then swapped between the balance pans (left to right and then right to left) without arresting the balance. Arresting the balance would change the effective rest point slightly and compromise the accuracy of the sensitivity measurement. The average effect of swapping the sensitivity weight (d_{av}) is calculated (the two figures should agree to better than 5%) and this value used to calculated the sensitivity as follows:

$$S = \frac{2 \times W_S}{d_{av}}$$

Where: d_{av} is the average scale difference
S is the measured sensitivity
W_S is the effective value of the sensitivity weight

The sensitivity should be checked at at least four points across the weighing range of the balance.

5.6.6 Repeatability of reading

The repeatability of reading can be checked by loading the balance and performing a series of releases calculating a rest point for each series. Apart from arrestment the balance remains undisturbed between releases making this measurement relatively insensitive to the user. This makes this test useful in providing a temporal measurement of the balance performance which is insensitive to external influences.

5.6.7 Repeatability of measurement

This provides a more accurate assessment of the balance's performance "in use". A series of repeated weighings are performed using the balance in its normal comparison mode (double weighing or substitution). The series of measured mass differences can be statistically analysed to give a measure of the balance performance.

The average measured mass difference can also be compared with the certified mass difference between the weights taking into account the uncertainties on these certified values.

5.7 Assessment of mechanical single pan balances

Single-pan mechanical balances have generally been replaced by electronic balances, which often offer better resolution and are easier to use. The range of tests that should be performed on these balances is only briefly described.

5.7.1 Visual inspection and mechanical check

- Check knife-edges and planes
- Ensure the released balance does not foul
- Adjust the zero of the balance

5.7.2 Drift

This will give an indication of the balance's stability and its sensitivity to changes in environmental conditions. It will also give an indication of how long the balance should be left to give a stable reading (the effectiveness of the damping). If the drift is linear it may be eliminated by the use of a suitable symmetrical weighing method (eg. ABA weighing).

5.7.3 Calibration of internal weights

Ideally the weights built into the balance should be removed and calibrated externally. If this is not possible they can be left in the balance and calibrated by dialling them up in combinations.

5.7.4 Effect of off centre loading (eccentricity)

The effect of off centre loading on the balance reading is assessed using a weight of nominal value equal to about half the balance capacity (to avoid mechanical damage to the balance). The weight is placed at the extremes of the pan and the results compared with the reading when the weight is placed centrally. A significant difference between the reading at the centre and extremes of the pan may indicate the balance is fouling. This test is identical to that described in more detail for a single pan electronic balance.

5.7.5 Hysteresis

This is checked by taking readings of increasing and decreasing load. Significant difference may indicate mechanical problems within the balance. This test is identical to that described in more detail for a single pan electronic balance.

5.7.6 Scale sensitivity (scale value)

This is checked by placing a weight on the pan equal to the full range value of the (optical) scale. For most single-pan mechanical balances adjustment of the scale sensitivity is possible. This test is identical to that described in more detail for a single pan electronic balance.

5.7.7 Scale linearity

The (optical) scale should be checked for linearity at regular intervals along its range using a calibrated set of (fractional) weights. This test is identical to that described in more detail for a single pan electronic balance.

5.7.8 Repeatability of reading

The repeatability of reading should be assessed by loading the balance (usually to maximum load) and taking a number of readings. The balance should be arrested and released between readings without otherwise disturbing it. This provides a measure of the balance performance, independent of external variables, and can thus be used as a repeatable check of the balance performance over time.

5.7.9 Repeatability of measurement

This represents a series of actual comparisons using the balance in its normal weighing mode. Statistical analysis of the repeatability of measurement provides a practical assessment of the balance performance under normal weighing conditions.

5.8 Assessment of electronic balances

Figure 8: Single-pan electronic balance

This type of balance is by far the most widely used and the simple principle of operation means that a full assessment takes the form of relatively few tests.

As with the other types of balance the manner in which the balance is assessed will depend on the way it is to be used in practice. Before starting the assessment of an electronic balance it should have been left switched on for a least an hour and preferably overnight (check the manufacturer's literature for details). Where appropriate the balance calibration facility should be used before starting the assessment.

5.8.1 Hysteresis

Hysteresis occurs when, for a given weight, the balance displays a different reading depending on whether the load is increasing or decreasing. It can be checked by:

- Incrementally increasing the load by adding weights one at a time, noting the balance reading after each addition
- Remove the weights in reverse order, noting the reading after each removal
- Compare the readings for the same load being applied to the balance

Significant deviation between the increasing and decreasing readings indicates a poorly adjusted or dirty balance.

5.8.2 Effect of off centre loading (eccentricity)

Electronic balances are generally more sensitive to off centre loading than either of the other types, as there is often little de-coupling between the balance pan and the sensing element. This effect is assessed by placing a weight, of value about half the capacity of the balance, at the extremes of the pan. Compare the results with the reading when the weight is placed centrally. Typical results are shown below.

Reading sequence	Centre	Right	Front	Left	Back
C > R > F > L > B	0.86 g	0.81 g	0.83 g	0.87 g	0.82 g
B > L > F > R > C	0.85 g	0.87 g	0.80 g	0.86 g	0.83 g
Mean reading	0.855 g	0.84 g	0.815 g	0.865 g	0.825 g
Reading relative to centre		-0.015 g	-0.04 g	+0.01 g	-0.03 g

The weight is placed at each position on the pan twice and the weighing scheme is symmetrical with time so any balance drift is calculated out.

5.8.3 Scale error and linearity

Scale error and linearity can be checked with a suitable set of calibrated mass standards. Measurements should be made at about ten equal steps across the range of the balance. If the balance is usually used at a particular load or an area of the scale is thought to be non-linear, additional linearity checks can be performed at that load.

5.8.4 Repeatability of reading

The repeatability of an electronic top pan balance can be assessed by repeated application of a mass standard. A set of ten weight applications should be measured at at least two loads (usually half and full load) and at any other load where the balance will be normally used.

5.8.5 Repeatability of measurement

A more relevant test for a balance that is always used as a comparator is to carry out repeated comparisons of a pair of weights using the same weighing scheme that would normally be used on the balance.

Generally the following data should be calculated for a repeatability assessment:

- Maximum difference between consecutive weighings
- Maximum difference between any weighings
- Standard deviation of the series (σ_{n-1})
- Standard deviation of the series accounting for drift

5.9 Periodicity of calibration

The period between balance calibrations depends on the intensity of use of the balance and on the type of measurements which are being performed. In general a full balance assessment should be undertaken annually and if any changes to the balance set up or location have occurred. If a balance is fitted with an internal calibration facility, this should be checked on a weekly basis or if changes are made to the balance set up.

6 Uncertainty in mass calibration

6.1 Introduction

When making measurements there is always an element of uncertainty in the result. We cannot know ‘true’ values – there are limitations in our knowledge and in the performance of the instruments we are using. Therefore a measurement is not complete without an estimate of the doubt that surrounds it (the uncertainty) and the confidence we have in that estimate. This chapter gives an introduction to the terminology and the main sources of uncertainty in the calibration of a weight of nominally 100 g, when using a comparator, plus a brief summary of the process for calculating the uncertainty and associated confidence level. All calculations are performed in accordance with the ISO Guide to the Expression of Uncertainty in Measurement [8] also known as the GUM.

6.2 The measurement

Although perhaps an obvious point, before starting it is worth confirming precisely what the measurements are aimed at determining. In this example it is the conventional mass of a weight and the following need to be considered:

- Which measurements and calculations will be required to enable you to establish the mass value and the uncertainty in its determination? For example, will you need to determine air density?
- How many measurements do you need to take? (The more measurements you take the more representative the mean (average) value becomes, although there is a reduction in benefit as the number of measurements increase beyond a certain point. Ten measurements is a common choice and statistically valid but not always

practical – eg for economic reasons. One way of dealing with this problem is to use some data from previous measurements to determine the performance of the balance and then take a smaller number of measurements for the particular calibration in hand. This is dealt with in paragraph 6.3.3 under *repeatability*.)

- How to take your measurements and calculate a mass value.

6.3 The uncertainty budget

Once you have determined the mass value you are ready to start calculating the uncertainty in the measurement. The following table is a typical layout for an uncertainty budget. It can be in the form of a computer spreadsheet – to make repeated calculations easier - or it can be a paper table completed by hand using a calculator. Each column in the table is dealt with separately below.

Symbol	Source of uncertainty	Value	Probability distribution	Divisor	Sensitivity coefficient	Standard uncertainty	v_i or v_{eff}

6.3.1 Symbol

The symbol used in to denote the input quantity or the influence factor.

6.3.2 Sources of uncertainty

The sources of the uncertainty are dependent on the measurement process and equation used - as defined in your procedures and by your laboratory environment. Here we consider the most common sources.

Each of the *input quantities* in the measurement equation used to calculate the mass value has an uncertainty. For example if the equation you are using is:

$$Wx = Ws + \Delta W + Ab$$

where Wx is the unknown mass
Ws is the mass of the standard
ΔW is the difference in the balance readings
Ab is the correction for air buoyancy

there is an uncertainty associated with each of the input quantities Ws, ΔW and Ab. The uncertainty in ΔW depends on uncertainty due to other influence factors:

- Rounding errors in the comparator readings (δI_d)
- The uncertainty due to less-than-perfect repeatability of the readings (W_R)
- The value associated with comparator linearity (δC)

Another influence factor to be considered is

- The drift of the standard with time (Ds)

Symbol	Source of uncertainty	Value	Probability distribution	Divisor	Sensitivity coefficient	Standard uncertainty	v_i or v_{eff}
Ws	Calibration of mass standard						
δC	Comparator linearity						
Ab	Air buoyancy						
Ds	Uncorrected drift of the standard						
δI_d	Digital rounding error						
W_R	Repeatability						

6.3.3 Values

The values associated with the sources of uncertainty are either measured, calculated or come from *a priori* (previous) knowledge.

In our example:

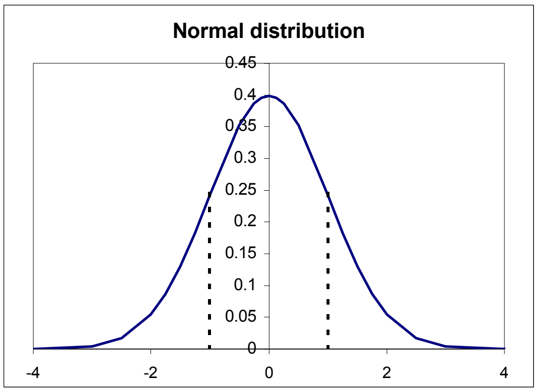
Symbol	**Uncertainty source**	**The value**
Ws	Calibration of mass standard	The uncertainty in the mass standard is taken from its calibration certificate.
δC	Comparator linearity	Estimated from previous measurements according to your procedures.
Ab	Air buoyancy	A calculated uncertainty based on the air buoyancy correction equation ie (Vs – Vx)(ρ_a – 1.2) (see paragraph 6.4) In this example the volumes have not been measured and the uncertainty in the volume difference is based on the values given in the OIML recommendation R111 (if the volumes are measured the uncertainty can be less) but the value of air density has been measured.
Ds	Uncorrected drift of the standard	The uncertainty quoted on the mass standard's calibration certificate will not include any contribution for drift in its mass value. The evaluation of this effect is normally the responsibility of the weight's owner as they are best placed to evaluate how much its mass changes between calibrations. Drift is usually determined by considering how much a particular artefact has changed its mass value over a recent period and extrapolating the figure to cover the period up to its next calibration. In this example no previous calibration knowledge is assumed and the uncertainty in the current mass value calibration is also used to estimate the limits of drift.
δI_d	Digital rounding error	Each reading is subject to a rounding error. It is taken to be ± half the resolution of the comparator. Such errors occur in the comparator reading of the standard mass and the unknown mass.
W_R	Repeatability	This is an uncertainty component which is a measure of the 'spread' of the repeated readings. It is estimated by determining the experimental standard deviation of the mean (see *Further reading*). In this example a previous evaluation of repeatability of the measurement process (from ten comparisons between a mass standard and an unknown mass) were used to establish a standard deviation of 0.00017 mg which was then divided by $\sqrt{n}$, where n is the number of readings in the current measurement – in this case three).

6.3.4 Probability distributions

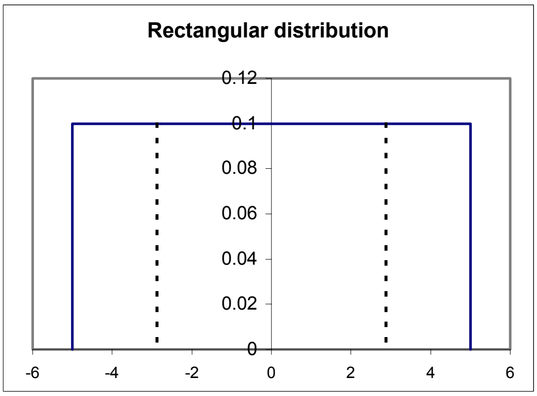
A probability distribution is a statistical description of how results behave. There are three distributions commonly used in mass uncertainty budgets: *normal* (sometimes called *Gaussian*), *rectangular* (sometimes called *uniform*) and *triangular*. The following graphs illustrate these distributions where the zero value on the horizontal axis represents the mean value of a number of readings, the vertical axis is the probability of a particular value occurring and the broken lines represent plus and minus one standard deviation (k=1) - encompassing approximately 68% of the measurement values (ie 68% of the area under each curve).

6.3.4.1 Normal distribution

This represents a group of measurements where the values are more likely to fall closer to the mean value than further away from it. Repeated measurements are an example of this type of distribution. The graph shows normally distributed data with a mean value of zero and a standard deviation of ±1.



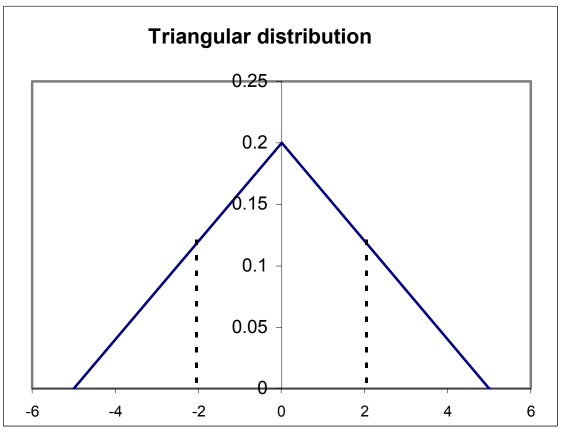
6.3.4.2 Rectangular distribution



This represents a group of measurements where the values are evenly spread between two limits and never fall outside these limits. An example is when using an assumed air buoyancy correction (as opposed to a measured or calculated value). The graph shows a rectangular distribution, again with a mean value of zero, but limits of ±5. In this case one standard deviation is ±2.89.

6.3.4.3 Triangular distribution

This is the distribution you get when adding two rectangular distributions. An example is the uncertainty due to rounding errors. The graph shows a triangular distribution, again with a mean value of zero and limits ±5. In this case one standard deviation is ±2.04.



6.3.5 Divisor

In order to eventually sum all the individual input quantities they must be quoted with the same confidence level. This is done by establishing a number by which the input quantity uncertainty value is divided to convert it to one standard deviation and is dependent on the distribution as shown in the table below.

Distribution	Divisor
Normal	1 or 2
Rectangular	√3
Triangular	√6

The normal distribution has a divisor of either 1 or 2 depending on the confidence level of the value quoted. For example on a certificate of calibration the uncertainty might be quoted as 'k = 1' (~68%) or 'k = 2' (~95%) in which case the divisor is the 'k' number. Other values are sometimes used, for example k = 3.

6.3.6 Sensitivity coefficient (c_i)

This is a multiplication factor which converts the uncertainty in the value of an input quantity to a corresponding uncertainty in the output quantity (it sometimes has to convert both *quantity* – such as temperature or pressure - to mass and also the right *units*). In the example being discussed all the input quantities are already expressed in the quantity *mass* and using the sub-unit *milligram* so the sensitivity coefficient is 1.

An explanation of the air buoyancy correction calculation is in paragraph 6.4.

6.3.7 Standard uncertainty in units of measurand, $u_i(W_i)$

In order to add all the components together we need them in the same units and the values for this column are simply calculated from

$$\text{Value} \div \text{Divisor} \times \text{Sensitivity coefficient } (c_i)$$

6.3.8 Degrees of freedom v_i

The number of degrees of freedom is "…in general the number of terms in a sum minus the number of constraints on the terms of the sum"[4]. Before considering this further it is necessary to first appreciate that measurement uncertainties are considered to fall into one of two categories, known as *Type A* and *Type B*.

- Type A uncertainties are those that are evaluated by statistical methods. For example, uncertainty due to less-than-perfect repeatability of a measurement can be reduced by calculating a mean value from several measurements.

- Type B uncertainties are evaluated by other means. They cannot be reduced by taking more measurements – the uncertainty quoted on a certificate of calibration cannot be made lower by repeatedly reading the certificate for example!

The degrees of freedom, v_i, for individual uncertainty contributions are given by:

Type A	v_i = n-1 where n is the number of measurements used to evaluate the type A contribution
Type B	v_i is usually taken to be infinite

6.3.9 Adding it all up

6.3.9.1 Combined standard uncertainty u(Wx)

To obtain an uncertainty in Wx, the mass value of the unknown weight, the components have to be added to obtain a *combined standard uncertainty*. As it is unlikely that all the errors will have been at their maximum value in any one measurement it is inappropriate to add them in arithmetically. The recognised way to address this issue is to arithmetically add the squares of the standard uncertainties and then take the square root of the result – this process is know variously as taking the *root sum of the squares* (RSS) or *quadrature summation*.

In the example we are considering the summation is:

$$u(Wx) = \sqrt{0.0250^2 + 0.0115^2 + 0.0216^2 + 0.0289^2 + 0.0020^2 + 0.0001^2}$$

$$= 0.0454 \text{ mg}$$

The resulting probability distribution will be a normal distribution unless one rectangular distribution is much larger than the other components.

6.3.9.2 Effective degrees of freedom v_{eff}

In general the effective degrees of freedom, $v_{eff,}$ will not need to be calculated if the type A uncertainty is less than half of the combined standard uncertainty, there is only one type A component and at least three measurements have been taken. Otherwise the effective degrees of freedom will have to be calculated to ensure that the k-factor of 2 will indeed give a confidence level of ~95%.

The effective degrees of freedom for the combined standard uncertainty will depend on the magnitude of the degrees of freedom for the type A contributions in relation to the type B. If the type B uncertainties are all taken to have infinite degrees of freedom the relationship is shown using the simplified Welch-Satterthwaite equation:

$$v_{eff} = \frac{u_c^4(y)}{\left(\frac{u_i^4(y)}{v_i}\right)}$$

where $u_c(y)$ is the combined standard uncertainty
$u_i(y)$ is the individual type A uncertainty contribution
v_i is the degree of freedom in $u_i(y)$

therefore in our example:

$$v_{eff} = \frac{0.0454^4}{\left(\frac{0.0001^4}{9}\right)} \quad \text{thus} \quad v_{eff} = 3.8\text{E}+11$$

In this example v_{eff} is a very large number which can be taken to be infinity. If this value had been less than 100 a k-factor would have been calculated from a distribution other than a normal distribution. More information about this can be found in the further reading list but for our example the k-factor is two.

6.3.9.3 Expanded uncertainty

The expanded uncertainty U(Wx) is the combined standard uncertainty u(Wx) multiplied by a k-factor which will give an uncertainty value with a confidence level of approximately 95%, in this case 2.

The final uncertainty budget for our example is shown below.

Symbol	Uncertainty source	Value ±mg	Probability distribution	Divisor	Sensitivity coefficient	Std. uncertainty ± mg	v_i or v_{eff}
Ws	Calibration of standard weight	0.0500	Normal	2	1	0.0250	∞
δC	Comparator linearity	0.0200	Rectangular	√3	1	0.0115	∞
Ab	Air buoyancy	0.0216	Normal	1	1	0.0216	∞
Ds	Uncorrected drift of the standard	0.0500	Rectangular	√3	1	0.0289	∞
δI_d	Digital rounding error	0.0050	Triangular	√6	1	0.0020	∞
W_R	Repeatability	0.0001	Normal	1	1	0.0001	9
u(Wx)	Combined standard uncertainty		Normal			0.0454	>500
U(Wx)	Expanded uncertainty		Normal k=2			0.0908	>500

6.4 Air buoyancy uncertainty budget

In order to calculate the uncertainty in the air buoyancy correction for entry into the main uncertainty budget an additional uncertainty budget has to be completed. Air buoyancy (Ab) is dependent on the volumes of the weights and the air density with the following relationship:

$$Ab = (Vs - Vx)(\rho_a - 1.2)$$

where (Vs - Vx) is the difference in volume between the standard and the unknown weight
(ρ_a - 1.2) is the difference between measured density of the air and the standard air density

There are two ways to calculate an uncertainty value for Ab, either working in relative values or calculating the sensitivity coefficient directly. In our example the volumes have not been measured so the value of (Vs - Vx) is taken to be the largest difference possible according to the OIML recommendations [1] when comparing E_2 and F_1 weights – that is 1.3 cm^3 with an uncertainty of ±1.3 cm^3. This uncertainty is treated as a rectangular distribution because the real value may lie anywhere between these limits and thus the standard uncertainty (u(V)) is equal to ±(1.3 ÷ √3). In this example the air density, ρ_a, has been measured as being 1.22 kg/m^3 with an uncertainty of ±10% - thus the uncertainty in (ρ_a – 1.2), that is u(ρ), is ±0.012 kg/m^3. Thus:

$$Ab = (1.3)(1.22 - 1.2) = 0.026$$

6.4.1.1 Relative uncertainty

The relative uncertainty in the air buoyancy, u(Ab)/Ab, is given by:

$$\frac{u(Ab)}{Ab}=\sqrt{\left(\frac{u(V)}{(Vx-Vs)}\right)^2+\left(\frac{u(\rho)}{(\rho_a-1.2)}\right)^2}$$

inserting the values gives:

$$u(Ab)=Ab\sqrt{\left(\frac{1.3\div\sqrt{3}}{1.3}\right)^2+\left(\frac{0.012}{1.22-1.2}\right)^2}$$

$$u(Ab)=0.0216$$

This method of calculation works well for an equation where the only operators are multiplication or division.

6.4.1.2 Partial differentiation

The other method, partial differentiation, sounds more complicated but is actually quite straightforward for this type of equation.

Using the same equation, Ab = (Vs - Vx)(ρ_a - 1.2), and the same values as above, we calculate the sensitivity coefficients - the numbers by which values of u(V), expressed here in cm^3, and u(ρ), expressed here in kg/m^3, should be multiplied to calculate their effect on the output quantity expressed in grams.

The partial derivative of a simple equation, such as the one we are looking at, is simply the multiplier for the term for which we wish to calculate the partial derivative. For example, if our equation is A = B × C then the partial derivative of B is C and the partial derivative C is B.

In our air density problem the partial derivative of (Vs - Vx) is (ρ_a - 1.2) and the partial derivative of (ρ_a - 1.2) is (Vs - Vx). In the correct terminology this is expressed as :

$$\frac{\partial(\mathrm{Ab})}{\partial(\mathrm{Vs}-\mathrm{Vx})}=(\rho_{\mathrm{a}}-1.2)=0.02$$

$$\frac{\partial(\mathrm{Ab})}{\partial(\rho_{\mathrm{a}}-1.2)}=(\mathrm{Vs}-\mathrm{Vx})=1.3$$

The values 0.02 and 1.3 are entered directly into the sensitivity coefficient column of the uncertainty budget and the remaining calculations are the same as for the main uncertainty budget.

Symbol	Source of uncertainty	Value ±mg	Probability distribution	Divisor	Sensitivity coefficient	Std. uncertainty ± mg	v_i or v_{eff}
Vs – Vx	Difference in volumes	1.3	Rectangular	√3	0.02	0.0150	∞
$\rho_a - 1.2$	Diff. in air density	0.012	Normal	1	1.3	0.0156	∞
Combined standard uncertainty			Normal			0.0216	∞

The combined standard uncertainty is the same as in the previous method.

Partial differentiation is more difficult when the equation is more complex.

6.5 Reporting the results

In general calculations should not be rounded until the final result is calculated. The uncertainty should be quoted to 2 significant figures and the result quoted to the same number of decimal places. In the worked example the result would be reported in the form:

100.000 71 g ±0.10 mg

and would be accompanied by a statement explaining how the uncertainty and its confidence level are calculated such as:

> *The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor k = 2, providing a level of confidence of approximately 95%. The uncertainty evaluation has been carried out in accordance with UKAS requirements.*

This statement was taken from the UKAS document [5].

The uncertainty has been rounded to 0.10 mg; uncertainties should always be rounded up rather than down to ensure that the value remains within the 95% confidence limit.

7 References

[1] International Organisation of Legal Metrology (OIML), International Recommendation No 111:1994 Weights of classes E1, E2, F1, M1, M2, M3.
[2] Giacomo P., Metrologia, **18**, 33-40, 1982
[3] Davis R.S., Metrologia, **29**, 67-70, 1992
[4] BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML. *Guide to the Expression of Uncertainty in Measurement*, International Organisation for Standardisation, Geneva. ISBN 92-67-10188-9
[5] UKAS publication M3003. *The Expression of Uncertainty and Confidence in Measurement*, 1997.
[6] Clarkson M.T., May B.J Metrologia, **38(2)**, 161-171, 2001
[7] Guide to the Expression of Uncertainty in Measurement, International Organization for Standardization, (Geneva, Switzerland), 1993