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Guidelines on the Calibration of Non-Automatic Weighing Instruments

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Calibration Guide

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GUIDELINES ON THE CALIBRATION OF NON-AUTOMATIC WEIGHING INSTRUMENTS

January 2009

Purpose

This document has been developed to improve harmonisation in calibration of Non-Automatic Weighing Instruments (NAWI). It gives advice to calibration laboratories to establish practical procedures.

The document contains detailed examples of the estimation of the uncertainty of measurements.

Authorship

This document was originally published by EA Committee 2 (Technical Activities), based on a draft of the Ad hoc Working Group "Mechanical measurements". It is revised and re-published by the EURAMET Technical Committee for Mass and Related Quantities.

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1 INTRODUCTION

Non-automatic weighing instruments are widely used to determine the quantity of a load in terms of mass. While for some applications specified by national legislation, they are subject to legal metrological control – i.e. type approval, verification etc. - there is an increasing need to have their metrological quality confirmed by calibration, e.g. where required by ISO 9001 or ISO/IEC 17025 standards.

2 SCOPE

This document contains guidance for the static calibration of self-indicating, non-automatic weighing instruments (hereafter called "instrument"), in particular for

1. measurements to be performed,
2. calculation of the measuring results,
3. determination of the uncertainty of measurement,
4. contents of calibration certificates.

The object of the calibration is the indication provided by the instrument in response to an applied load. The results are expressed in units of mass. The value of the load indicated by the instrument will be affected by local gravity, the load's temperature and density, and the temperature and density of the surrounding air.

The uncertainty of measurement depends significantly on properties of the calibrated instrument itself, not only on the equipment of the calibrating laboratory; it can to some extent be reduced by increasing the number of measurements performed for a calibration. This guideline does not specify lower or upper boundaries for the uncertainty of measurement.
It is up to the calibrating laboratory and the client to agree on the anticipated value of the uncertainty of measurement which is appropriate in view of the use of the instrument and in view of the cost of the calibration.

While it is not intended to present one or few uniform procedures the use of which would be obligatory, this document gives general guidance for the establishing of calibration procedures the results of which may be considered as equivalent within the SIM Member Organisations.

Any such procedure must include, for a limited number of test loads, the determination of the error of indication and of the uncertainty of measurement assigned to these errors. The test procedure should as closely as possible resemble the weighing operations that are routinely being performed by the user – e.g. weighing discrete loads, weighing continuously upwards and/or downwards, use of tare balancing function.

The procedure may further include rules how to derive from the results advice to the user of the instrument with regard to the errors, and assigned uncertainty of measurement, of indications which may occur under normal conditions of use of the instrument, and/or rules on how to convert an indication obtained for a weighed object into the value of mass or conventional value of mass of that object.

The information presented in this guideline is intended to serve, and should be observed by,

1. bodies accrediting laboratories for the calibration of weighing instruments,
2. laboratories accredited for the calibration of non-automatic weighing instruments,
3. testhouses, laboratories, or manufacturers using calibrated non-automatic weighing instruments for measurements relevant for the quality of production subject to QM requirements (e.g. ISO 9000 series, ISO 10012, ISO/IEC 17025).

A summary of the main terms and equations used in this document is given in Appendix D2.

3 TERMINOLOGY AND SYMBOLS

The terminology used in this document is mainly based on existing documents:

- GUM [1] for terms related to the determination of results and the uncertainty of measurement,
- OIML R111 [4] for terms related to the standard weights,
- OIML R76 [2] (or EN 45501 [3]) for terms related to the functioning, to the construction, and to the metrological characterisation of non-automatic weighing instruments.
- VIM [8] for terms related to the calibration.

Such terms are not explained in this document, but where they first appear, references will be indicated.

Symbols whose meaning is not self-evident, will be explained where they are first used. Those that are used in more than one section are collected in Appendix D1.

4 GENERAL ASPECTS OF THE CALIBRATION

4.1 Elements of the calibration

Calibration consists of

1. applying test loads to the instrument under specified conditions,
2. determining the error or variation of the indication, and
3. estimating the uncertainty of measurement to be attributed to the results.

4.1.1 Range of calibration

Unless requested otherwise by the client, a calibration extends over the full weighing range [3] from Zero to the maximum capacity Max. The client may specify a certain part of a weighing range, limited by a minimum load Min' and the largest load to be weighed Max', or individual nominal loads, for which he requests calibration.

On a multiple range instrument [3], the client should identify which range(s) shall be calibrated. The preceding paragraph applies to each range separately.

4.1.2 Place of calibration

Calibration is normally performed on the site where the instrument is being used.

If an instrument is moved to another location after the calibration, possible effects from

1. difference in local gravity acceleration,
2. variation in environmental conditions,
3. mechanical and thermal conditions during transportation.

are likely to alter the performance of the instrument and may invalidate the calibration. Moving the instrument after calibration should therefore be avoided, unless immunity to these effects of a particular instrument, or type of instrument has been clearly demonstrated. Where this has not been demonstrated, the calibration certificate should not be accepted as evidence of traceability.

4.1.3 Preconditions, preparations

Calibration should not be performed unless

1. the instrument can be readily identified,
2. all functions of the instrument are free from effects of contamination or damage, and functions essential for the calibration operate as intended,
3. presentation of weight values is unambiguous and indications, where given, are easily readable,
4. the normal conditions of use (air currents, vibrations, stability of the weighing site etc.) are suitable for the instrument to be calibrated,
5. the instrument is energized prior to calibration for an appropriate period, e.g. as long as the warm-up time specified for the instrument, or as set by the user,
6. the instrument is levelled, if applicable,
7. the instrument has been exercised by loading approximately up to the largest test load at least once, repeated loading is advised.

Instruments that are intended to be regularly adjusted before use should be adjusted before the calibration, unless otherwise agreed with the client. Adjustment should be performed with the means that are normally applied by the client, and following the manufacturer’s instructions where available.

As far as relevant for the results of the calibration, the status of software settings which can be altered by the client should be noted.

Instruments fitted with an automatic zero-setting device or a zero-tracking device [3] should be calibrated with the device operative or not, as set by the client.

For on site calibration the user of the instrument should be asked to ensure that the normal conditions of use prevail during the calibration. In this way disturbing effects such as air currents, vibrations, or inclination of the measuring platform will, so far as is possible, be inherent to the measured values and will therefore be included in the determined uncertainty of measurement.

4.2 *Test load and indication*

4.2.1 Basic relation between load and indication

In general terms, the indication of an instrument is proportional to the force exerted by an object of mass m on the load receptor:

$$I \sim mg\left(1-\rho_a/\rho\right) \qquad (4.2.2\text{-}1)$$

with g local gravity acceleration

ρ_a density of the surrounding air

ρ density of the object

The terms in the brackets account for the reduction of the force due to the air buoyancy of the object.

4.2.2 Effect of air buoyancy

It is state of the art to use standard weights that have been calibrated to the conventional value of mass m_c [1], for the adjustment and/or the calibration of weighing instruments. The adjustment is performed such that effects of g and of the actual buoyancy of the standard weight m_{cs} are included in the adjustment factor. Therefore, at the moment of the adjustment the indication I_s is:

$$I_s = m_{cs} \qquad (4.2.2\text{-}1)$$

This adjustment is, of course, performed under the conditions characterized by the actual values of g_s, $\rho_s \neq \rho_c$, and $\rho_{as} \neq \rho_0$, identified by the suffix "s", and is valid only under these conditions. For another body with $\rho \neq \rho_s$, weighed on the same instrument but under different conditions: $g \neq g_s$ y $\rho_a \neq \rho_{as}$ the indication is in general (neglecting terms of 2nd or higher order):

$$I = m_c\left(g/g_s\right)\left\{1-\left(\rho_a-\rho_0\right)\left(1/\rho-1/\rho_s\right)-\left(\rho_a-\rho_{as}\right)/\rho_s\right\} \qquad (4.2.2\text{-}3)$$

If the instrument is not displaced, there will be no variation of g, so $g/g_s = 1$. This is assumed hereafter.

The formula simplifies further in situations where some of the density values are equal:-

a) weighing a body in the reference air density: $\rho_a = \rho_0$, then

$$I = m_c\left\{1-\left(\rho_a-\rho_{as}\right)/\rho_s\right\} \qquad (4.2.2\text{-}4)$$

---

[1] The conventional value of mass m_c of a body has been defined in [4] as the numerical value of mass m of a weight of reference density ρ_c = 8000 kg/m³ which balances that body at 20 °C in air of density ρ_0:

$$m_c = m\left\{\left(1-\rho_0/\rho\right)/\left(1-\rho_0/\rho_c\right)\right\} \qquad (4.2.2\text{-}2)$$

with ρ_0 = 1,2 kg/m³ = reference value of the air density

b) weighing a body of the same density as the adjustment weight:
$\rho = \rho_s$, then again (as in case a))

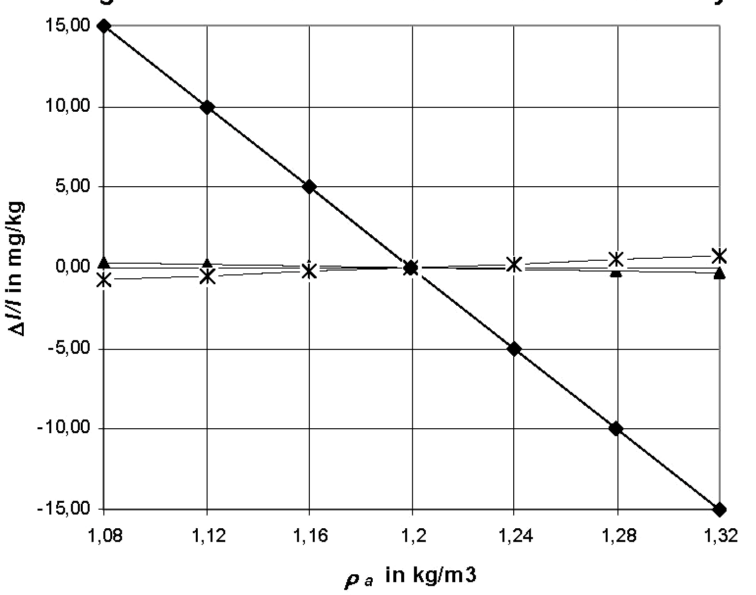
$$I = m_c\{1 - (\rho_a - \rho_{as})/\rho_s\} \quad (4.2.2\text{-}5)$$

c) weighing in the same air density as at the time of adjustment:
$\rho_a = \rho_{as}$, then

$$I = m_c\{1 - (\rho_a - \rho_0)(1/\rho - 1/\rho_s)\} \quad (4.2.2\text{-}6)$$

Figure 4.2-1 shows examples for the magnitude of the relative changes $\Delta I / I_s = (I - I_s)/I_s$ for an instrument adjusted with standard weights of $\rho_s = \rho_c$, when calibrated with standard weights of different but typical density.

Figure 4.2-1 Variation of indication due to air buoyancy



Line ▲ is valid for a body of $\rho =$ 7 810 kg/m³, weighed in $\rho_a = \rho_{as}$

Line ✕ is valid for a body of $\rho =$ 8 400 kg/m³, weighed in $\rho_a = \rho_{as}$

Line ◆ is valid for a body of $\rho = \rho_s = \rho_c$ after adjustment in $\rho_{as} = \rho_0$

It is obvious that under these conditions, a variation in air density has a far greater effect than a variation in the body's density.

Further information is given on air density in Appendix A, and on air buoyancy related to standard weights in Appendix E.

4.2.3 Effects of convection

Where weights have been transported to the calibration site they may not have the same temperature as the instrument and its environment. Two phenomena should be noted in this case:

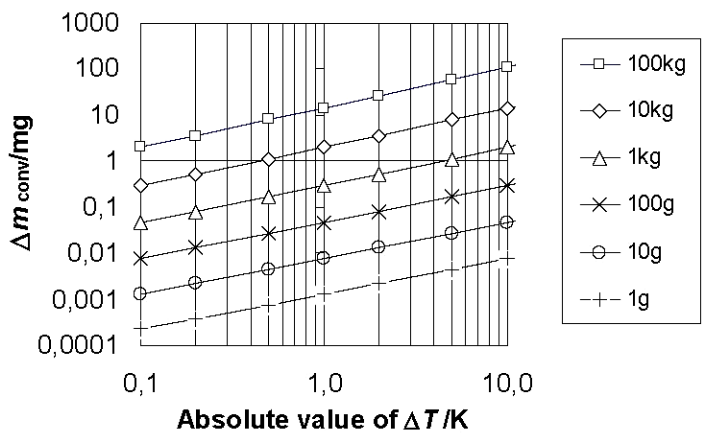
An initial temperature difference ΔT_0 may be reduced to a smaller value ΔT by acclimatisation over a time Δt; this occurs faster for smaller weights than for larger ones.

When a weight is put on the load receptor, the actual difference ΔT will produce an air flow about the weight leading to parasitic forces which result in an apparent change Δm_{conv} on its mass. The sign of Δm_{conv} is normally opposite to the sign of ΔT, its value is greater for large weights than for small ones.

The relations between any of the quantities mentioned: ΔT_0, Δt, ΔT, m and Δm_{conv} are nonlinear, and they depend on the conditions of heat exchange between the weights and their environment – see [6].

Figure 4.2-2 gives an impression of the magnitude of the apparent change in mass in relation to a temperature difference, for some selected weight values.

Figure 4.2-2 Effect of convection



This effect should be taken into account by either letting the weights accommodate to the extent that the remaining change Δm_{conv} is negligible in view of the uncertainty of the calibration required by the client, or by considering the possible change of indication in the uncertainty budget. The effect may be significant for weights of high accuracy, e.g. for weights of class E_2 or F_1 in R 111 [4].

More detailed information is given in Appendix F.

4.2.4 Reference value of mass

The general relations (4.2.2-3) to (4.2.2-6) apply also if the "body weighed" is a standard weight used for calibration.

To determine the errors of indication of an instrument, standard weights of known conventional value of mass m_{cCal} are applied. Their density ρ_{Cal} is normally different from the reference value ρ_c and the air density ρ_{aCal} at the time of calibration is normally different from ρ_0.

The error E of indication is

$$E = I - m_{ref} \qquad (4.2.4\text{-}1)$$

where m_{ref} is a conventional true value of mass, further called reference value of mass. Due to effects of air buoyancy, convection, drift and others which may lead to minor correction terms δm_x, m_{ref} is not exactly equal to m_{cCal}:

$$m_{ref} = m_{cCal} + \delta m_B + \delta m_{conv} + \delta m_D + \delta m... \qquad (4.2.4\text{-}2)$$

The correction for air buoyancy δm_B is affected by values of ρ_s and ρ_{as}, that were valid for the adjustment but are not normally known. It is assumed that weights of the reference density $\rho_s = \rho_c$ have been used. (4.2.2-3) then gives the general expression for the correction

$$\delta m_B = -m_{cCal}\left[(\rho_{aCal} - \rho_0)(1/\rho_{Cal} - 1/\rho_c) + (\rho_{aCal} - \rho_{as})/\rho_c\right] \qquad (4.2.4\text{-}3)$$

For the air density ρ_{as} two situations are considered:

A The instrument has been adjusted immediately before the calibration, so $\rho_{as} = \rho_{aCal}$. This simplifies (4.2.4-3) to:

$$\delta m_B = -m_{cCal}(\rho_{aCal} - \rho_0)(1/\rho_{Cal} - 1/\rho_c) \qquad (4.2.4\text{-}4)$$

B The instrument has been adjusted independent of the calibration, in unknown air density ρ_{as} for which a reasonable assumption should be made.

B1 For on-site calibrations, ρ_{as} may be expected to be similar to ρ_{aCal}, with the possible difference $\delta\rho_{as} = \rho_{aCal} - \rho_{as}$. (4.2.4-3) is then modified to

$$\delta m_B = -m_{cCal}\left[(\rho_{aCal} - \rho_0)(1/\rho_{Cal} - 1/\rho_c) + \delta\rho_{as}/\rho_c\right] \qquad (4.2.4\text{-}5)$$

B2 A simple, straightforward assumption could be $\rho_{as} = \rho_0$, then

$$\delta m_B = -m_{cCal}(\rho_{aCal} - \rho_0)/\rho_{Cal} \qquad (4.2.4\text{-}6)$$

See also Appendices A and E for further information.

The other correction terms are dealt with in section 7.
The suffix "*Cal*" will from now on be omitted unless where necessary to avoid confusion.

4.3 Test loads

Test loads should preferably consist of standard weights that are traceable to the SI unit of mass. Other test loads may be used, however, for tests of a comparative nature – e.g. test with eccentric loading, repeatability test – or for the mere loading of an instrument – e.g. preloading, tare load that is to be balanced, substitution load.

4.3.1 Standard weights

The traceability of weights to be used as standards shall be accomplished by calibration[2] consisting of

1. determination of the actual conventional value of mass m_c and/or the correction δm_c to its nominal value m_N : $\delta m_c = m_c - m_N$, together with the expanded uncertainty of the calibration U_{95}, or
2. confirmation that m_c is within specified maximum permissible errors mpe : $m_N - (mpe - U_{95}) < m_c < m_N + (mpe - U_{95})$

The standards should further satisfy the following requirements to the extent as appropriate in view of their accuracy:

3. density ρ_s sufficiently close to ρ_C = 8 000 kg/m³
4. surface finish suitable to prevent a change in mass through contamination by dirt or adhesion layers
5. magnetic properties such that interaction with the instrument to be calibrated is minimized.

Weights that comply with the relevant specifications of the International Recommendation OIML R 111 [4] should satisfy all these requirements.

The maximum permissible errors, or the uncertainties of calibration of the standard weights shall be compatible with the scale interval *d* [3] of the instrument and/or the needs of the client with regard to the uncertainty of the calibration of his instrument.

4.3.2 Other test loads

For certain applications mentioned in 4.3, 2nd sentence, it is not essential that the conventional value of mass of a test load is known. In these cases, loads other than standard weights may be used, with due consideration of the following:

1. shape, material, composition should allow easy handling,
2. shape, material, composition should allow the position of the centre of gravity to be readily estimated,
3. their mass must remain constant over the full period they are in use for the calibration,
4. their density should be easy to estimate,
5. loads of low density (e.g. containers filled with sand or gravel), may require special attention in view of air buoyancy.

Temperature and barometric pressure may need to be monitored over the full period the loads are in use for the calibration.

[2] ILAC-P 10-2002, nr. 2(b): Traceability shall be derived, where possible,...“from a calibration laboratory that can demonstrate competence, measurement capability and traceability with appropriate measurement uncertainty, e.g. an accredited calibration laboratory...“ and Note 3: „It is recognised by ILAC that in some economies calibrations performed by verifying authorities appointed under their economies’ Legal metrology frameworks are also accepted.“

4.3.3 Use of substitution loads

A test load the conventional value of mass of which is essential, should be made up entirely of standard weights. But where this is not possible, any other load which satisfies 4.3.2 may be used for substitution. The instrument under calibration is used as comparator to adjust the substitution load L_{sub} so that it brings about approximately the same indication I as the corresponding load L_{St} made up of standard weights.

A first test load L_{T1} made up of standard weights m_{c1} is indicated as:

$$I(L_{St}) = I(m_{c1}) \tag{4.3.3-1}$$

After removing L_{St} a substitution load L_{sub1} is put on and adjusted to give approximately the same indication:

$$I(L_{sub1}) \approx I(m_{c1}) \tag{4.3.3-2}$$

so that

$$L_{sub1} = m_{c1} + I(L_{sub1}) - I(m_{c1}) = m_{c1} + \Delta I_1 \tag{4.3.3-3}$$

The next test load L_{T2} is made up by adding m_{c1}

$$L_{T2} = L_{sub1} + m_{c1} = 2m_{c1} + \Delta I_1 \tag{4.3.3-4}$$

m_{c1} is again replaced by a substitution load of ≈ L_{sub1} with adjustment to ≈ $I(L_{T2})$.

The procedure may be repeated, to generate test loads *LT3, ...LTn* :

$$L_{Tn} = nm_{c1} + \Delta I_1 + \Delta I_2 + \ldots + \Delta I_{n-1} \tag{4.3.3-5}$$

The value of L_{Tn} is taken as the conventional value of mass m_c of the test load.

With each substitution step however, the uncertainty of the total test load increases substantially more than if it were made up of standard weights only, due to the effects of repeatability and resolution of the instrument. – cf. also 7.1.2.6[3].

4.4 Indications

4.4.1 General

Any indication I related to a test load is basically the difference of the indications I_L under load and I_0 at no-load:

$$I = I_L - I_0 \tag{4.4.1-1}$$

It is to be preferred to record the no-load indications together with the load

[3] Example: for an instrument with *Max* = 5000 kg, *d* = 1 kg, the standard uncertainty of 5 t standard weights may be 200 g, while the standard uncertainty of a test load made up of 1 t standard weights and 4 t substitution load, will be about 2 kg

indications for any test measurement. However, recording the no-load indications may be redundant where a test procedure calls for the setting to zero of any no-load indication which is not = zero of itself, before a test load is applied.

For any test load, including no load, the indication I of the instrument is read and recorded only when it can be considered as being stable. Where high resolution of the instrument, or environmental conditions at the calibration site prevent stable indications, an average value should be recorded together with information about the observed variability (e.g. spread of values, unidirectional drift).

During calibration tests, the original indications should be recorded, not errors or variations of the indication.

4.4.2 Resolution

Indications are normally obtained as integer multiples of the scale interval *d*.

At the discretion of the calibration laboratory and with the consent of the client, means to obtain indications in higher resolution than in *d* may be applied, e.g. where compliance to a specification is checked and smallest uncertainty is desired. Such means may be:

1. switching the indicating device to a smaller scale interval *dT* < *d* ("service mode").
   In this case, the indication I_x is then obtained as integer multiple of *dT*.
2. applying small extra test weights in steps of $d_T = d/5$ or $d/10$ to determine more precisely the load at which an indication changes unambiguously from I' to $I' + d$. ("changeover point method"). In this case, the indication I' is recorded together with the amount ΔL of the n additional small test weights necessary to increase I' by one d.

The indication I_L is

$$I_L = I' + d/2 - \Delta L = I' + d/2 - nd_T \qquad (4.4.2\text{-}1)$$

Where the changeover point method is applied, it is advised to apply it for the indications at zero as well where these are recorded.

5 MEASUREMENT METHODS

Tests are normally performed to determine

- the repeatability of indications,
- the errors of indications,
- the effect of eccentric application of a load on the indication.

A Calibration Laboratory deciding on the number of measurements for its routine calibration procedure, should consider that in general, a larger number of measurements tends to reduce the uncertainty of measurement but to increase the cost.

Details of the tests performed for an individual calibration may be fixed by agreement of the client and the Calibration Laboratory, in view of the normal use of the instrument. The parties may also agree on further tests or checks which may assist in evaluating the performance of the instrument under special conditions of use. Any such agreement should be consistent with the minimum numbers of tests as specified in the following sections.

5.1 Repeatability test

The test consists in the repeated deposition of the same load on the load receptor, under identical conditions of handling the load and the instrument, and under constant test conditions, both as far as possible.

The test load(s) need not be calibrated nor verified, unless the results serve for the determination of errors of indication as per 5.2. The test load should, as far as possible, consist of one single body.

The test is performed with at least one test load L_T which should be selected in a reasonable relation to Max and the resolution of the instrument, to allow an appraisal of the intrument's performance. For instruments with a constant scale interval *d* a load of $0{,}5Max \le L_T \le Max$ is quite common; this is often reduced for instruments where $L_T > 0{,}5Max$ would amount to several 1000 kg. For multi-interval instruments [3] a load close to Max_1 may be preferred. A special value of L_T may be agreed between the parties where this is justified in view of a specific application of the instrument.

The test may be performed at more than one test point, with test loads L_{Tj}, $1 \le j \le k_L$ with k_L = number of test points.

Prior to the test, the indication is set to zero. The load is to be applied at least 5 times, and at least 3 times where L_T ≥ 100 kg.

Indications I_{Li} are recorded for each deposition of the load. After each removal of the load, the indication should at least be checked for showing zero, and may be reset to zero if it does not; recording of the no-load indications I_{0i} is advisable as per 4.4.1. In addition, the status of the zero device if fitted is recorded.

5.2 Test for errors of indication

This test is performed with $k_L \geq 5$ different test loads L_{Tj}, $1 \leq j \leq k_L$, distributed fairly evenly over the normal weighing range[4] or at individual test points agreed upon as per 4.1.2.

The purpose of this test is an appraisal of the performance of the instrument over the whole weighing range.

Where a significantly smaller range of calibration has been agreed to, the number of test loads may be reduced accordingly, provided there are at least 3 test points including Min' and Max', and the difference between two consecutive test loads is not greater than $0{,}15Max$.

It is necessary that test loads consist of appropriate standard weights, or of substitution loads as per 4.3.3.

Prior to the test, the indication is set to zero. The test loads L_{Tj} are normally applied once in one of these manners:

1. increasing by steps with unloading between the separate steps – corresponding to the majority of uses of the instruments for weighing single loads,
2. continuously increasing by steps – similar to 1; may include creep effects in the results, reduces the amount of loads to be moved on and off the load receptor as compared to 1,
3. continuously increasing and decreasing by steps – procedure prescribed for verification tests in [3], same comments as for 2,
4. continuously decreasing by steps starting from Max - simulates the use of an instrument as hopper weigher for subtractive weighing, same comments as for 2.

On multi-interval instruments – see [3], the methods above may be modified for load steps smaller than Max, by applying increasing and/or decreasing tare loads, operating the tare balancing function, and applying a test load of close to but not more than Max_1 to obtain indications with *d1*.

Further tests may be performed to evaluate the performance of the instrument under special conditions of use, e.g. the indication after a tare balancing operation, the variation of the indication under a constant load over a certain time, etc.

The test, or individual loadings, may be repeated to combine the test with the repeatability test under 5.1.

Indications I_{Lj} are recorded for each load. After each removal of a load, the indication should at least be checked for showing zero, and may be reset to zero if it does not; recording of the no-load indications I_{0j} as per 4.4.1.

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[4] Examples for target values:

k_L = 5: zero or *Min*; *0,25* Max; *0,5 Max*; *0,75 Max; Max*. Actual test loads may deviate from the target value up to 0,1 *Max*, provided the difference between consecutive test loads is at least *0,2 Max*.

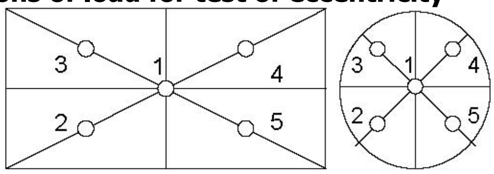
k_L = 11: zero or *Min*, 10 steps of *0,1 Max* up to *Max*. Actual test loads may deviate from the target value up to *0,05 Max*, provided the difference between consecutive test loads is at least *0,08 Max*.

5.3 *Eccentricity test*

The test consists in placing a test load L_{ecc} in different positions on the load receptor in such a manner that the centre of gravity of the load takes the positions as indicated in Figure 5.3-1 or equivalent positions, as closely as possible.

Fig. 5.3-1 Positions of load for test of eccentricity

1. Centre
2. Front left
3. Back left
4. Back right
5. Front right



The test load L_{ecc} should be at least $Max/3$, or at least $Min' + (Max' - Min')/3$ for a reduced weighing range. Advice of the manufacturer, if available, and limitations that are obvious from the design of the instrument should be considered – e.g. see OIML R76 [2] (or EN 45501 [3]) for weighbridges.

The test load need not be calibrated nor verified, unless the results serve for the determination of errors of indication as per 5.2.

Prior to the test, the indication is set to Zero. The test load is first put on position 1, is then moved to the other 4 positions in arbitrary order, and may at last be again put on position 1.

Indications I_{Li} are recorded for each position of the load. After each removal of the load, the zero indication should be checked and may, if appropriate, be reset to zero; recording of the no-load indications I_{0j} as per 4.4.1.

5.4 *Auxiliary measurements*

The following additional measurements or recordings are recommended, in particular where a calibration is intended to be performed with the lowest possible uncertainty.

In view of buoyancy effects – cf. 4.2.2:

The air temperature in reasonable vicinity to the instrument should be measured, at least once during the calibration. Where an instrument is used in a controlled environment, the span of the temperature variation should be noted, e.g. from a temperature graph, from the settings of the control device etc.

Barometric pressure or, by default, the altitude above sea-level of the site may be useful.

In view of convection effects – cf 4.2.3:

Special care should be taken to prevent excessive convection effects, by observing a limiting value for the temperature difference between standard weights and instrument, and/or recording an acclimatisation time that has been accomplished. A thermometer kept inside the box with standard weights may be helpful, to check the temperature difference.

In view of effects of magnetic interaction:

On high resolution instruments a check is recommended to see if there is an observable effect of magnetic interaction. A standard weight is weighed together with a spacer made of non-metallic material (e.g. wood, plastic), the spacer being placed on top or underneath the weight to obtain two different indications.

If the difference of these two indications is different from zero, this should be mentioned as a warning in the calibration certificate.

6 MEASUREMENT RESULTS

The formulae in chapters 6 and 7 are intended to serve as elements of a standard scheme for an equivalent evaluation of the results of the calibration tests. Where they are being applied unchanged as far as applicable, no further description of the evaluation is necessary.

It is not intended that all of the formulae, symbols and/or indices are used for presentation of the results in a Calibration Certificate.

The definition of an indication I as given in 4.4 is used in this section.

6.1 Repeatability

From the *n* indications I_{ji} for a given test load L_{Tj}, the standard deviation s_j is calculated

$$s\left(I_j\right) = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(I_{ji} - \bar{I}_j\right)^2} \quad (6.1\text{-}1)$$

with

$$\bar{I}_j = \frac{1}{n}\sum_{i=1}^{n} I_{ji} \quad (6.1\text{-}2)$$

Where only one test load has been applied, the index j may be omitted.

6.2 Errors of indication

6.2.1 Discrete values

For each test load L_{Ti}, the error of indication is calculated as follows:

$$E_j = I_j - m_{refj} \quad (6.2\text{-}1a)$$

Where an indication I_j is the mean of more than one reading, I_j is understood as being the mean value as per (6.1-2).

m_{ref} is the reference weight or "true value" of the load. – cf. 4.3.1, 4.3.3.

The reference weight is

either the nominal value m_N of the load,

$$m_{refj} = m_{Nj} \quad (6.2\text{-}2)$$

or its actual value m_c

$$m_{refj} = m_{cj} = \left(m_{Nj} + \delta m_{cj}\right) \quad (6.2\text{-}3)$$

Where a test load was made up of more than 1 weight, m_{Nj} is replaced by $\left(\sum m_N\right)_j$ and δm_{cj} is replaced by $\left(\sum \delta m_c\right)_j$ in the formulae above.

Where an error and/or indication is listed or used further in relation to the test load, it should always be presented in relation to the nominal value m_N of the load, even if the actual value of mass of the test load has been used. In such a case, the error remains unchanged, while the indication is modified by

$$I(m_N) = I'(m_c) - \delta m_c \tag{6.2-4}$$

with I' being the (interim) indication determined when m_c was applied.

(6.2-1a) then takes the form

$$E_j = I_j - m_{Nj} = \left(I'_j - \delta m_{cj}\right) - m_{Nj} \tag{6.2-1b}$$

6.2.2 Characteristic of the weighing range

In addition, or as an alternative to the discrete values I_j, E_j, a characteristic, or calibration curve may be determined for the weighing range, which allows to estimate the error of indication for any indication I within the weighing range.

A function

$$E_{appr} = f(I) \tag{6.2-5}$$

may be generated by an appropriate approximation which should in general, be based on the "least squares" approach:

$$\sum v_j^2 = \sum \left(f\left(I_j\right) - E_j\right)^2 = \text{minimum} \tag{6.2-6}$$

with
v_j = residual
f = approximation function

The approximation should further
take account of the uncertainties $u(E_j)$ of the errors,
use a model function that reflects the physical properties of the instrument, e.g. the form of the relation between load and its indication $I = g(L)$,
include a check whether the parameters found for the model function are mathematically consistent with the actual data.

It is assumed that for any m_{Nj} the error E_j remains the same if the actual indication I_j is replaced by its nominal value I_{Nj}. The calculations to evaluate (6.2-6) can therefore be performed with the data sets m_{Nj}, E_j, or I_{Nj}, E_j.

Appendix C offers advice for the selection of a suitable approximation formula and for the necessary calculations.

6.3 Effect of eccentric loading

From the indications I_i obtained in the different positions of the load as per 5.3, the differences ΔI_{ecc} are calculated

$$\Delta I_{ecci} = I_i - I_1 \tag{6.3-1}$$

Where the test load consisted of standard weight(s), the errors of indication may be calculated instead:

$$E_{ecci} = I_i - m_N \tag{6.3-2}$$

7 UNCERTAINTY OF MEASUREMENT

In this section and the ones that follow, there are terms of uncertainty assigned to small corrections, which are proportional to a specified mass value or to a specified indication. For the quotient of such an uncertainty divided by the related value of mass or indication, the abbreviated notation $\hat{w}$ will be used.
Example: let

$$u(\delta m_{corr}) = mu(corr) \tag{7-1}$$

with the dimensionless term $u(corr)$, then

$$\hat{w}(m_{corr}) = u(corr) \tag{7-2}$$

Accordingly, the related variance will be denoted by $\hat{w}^2(m_{corr})$ and the related expanded uncertainty by $\hat{W}(m_{corr})$.

7.1 Standard uncertainty for discrete values

The basic formula for the calibration is

$$E = I - m_{ref} \tag{7.1-1}$$

with the variances

$$u^2(E) = u^2(I) + u^2(m_{ref}) \tag{7.1-2}$$

Where substitution loads are employed – see 4.3.3 - m_{ref} is replaced by L_{Tn} in both expressions.

The terms are expanded further hereafter.

7.1.1 Standard uncertainty of the indication

To account for sources of variability of the indication, (4.4.1-1) is amended by correction terms δI_{xx} as follows:

$$I = I_L + \delta I_{digL} + \delta I_{rep} + \delta I_{ecc} - I_0 - \delta I_{dig0} \tag{7.1.1-1}$$

All these corrections have the expectation value zero. Their standard uncertainties are:

7.1.1.1 δI_{dig0} accounts for the rounding error of no-load indication. Limits are $\pm d_0/2$ or $\pm d_T/2$ as applicable; rectangular distribution is assumed, therefore

$$u(\delta I_{dig0}) = d_0/(2\sqrt{3}) \tag{7.1.1-2a}$$

or

$$u(\delta I_{dig0}) = d_T/(2\sqrt{3}) \tag{7.1.1-2b}$$

respectively.

Note 1: cf. 4.4.2 for significance of d_T.

Note 2: on an instrument which has been type approved to OIML R76 [2] (or EN 45501 [3]), the rounding error of a zero indication after a zero-setting or tare balancing operation is limited to $\pm d_0/4$, therefore

$$u(\delta I_{dig0}) = d_0/(4\sqrt{3}) \tag{7.1.1-2c}$$

7.1.1.2 δI_{digL} accounts for the rounding error of indication at load. Limits are $\pm d_I/2$ or $\pm d_T/2$ as applicable; rectangular distribution to be assumed, therefore

$$u(\delta I_{digL}) = d_I/2\sqrt{3} \quad \text{(7.1.1-3a}$$

or

$$u(\delta I_{digL}) = d_T/2\sqrt{3} \tag{7.1.1-3b}$$

Note: on a multi-interval instrument, d_I varies with I!

7.1.1.3 δI_{rep} accounts for the error due to imperfect repeatability; normal distribution is assumed, estimated

$$u(\delta I_{rep}) = s(I_j) \tag{7.1.1-5}$$

with $s(I_j)$ as per 6.1.

Where an indication I_j is the mean of *n* readings, the corresponding standards uncertainty is

$$u(\delta I_{rep}) = s(I_j)/\sqrt{n} \tag{7.1.1-6}$$

Where only one repeatability test has been performed, this standard deviation may be considered as being representative for all indications of the instrument in the weighing range considered.

Where several s_j ($s_j = s(I_j)$ in abbreviated notation) have been determined with different test loads, the greater value of s_j for the two test points enclosing the indication whose error has been determined, should be used.

Where it can be established that the values of s_j determined at different test loads L_{Tj}, are in functional relation to the load, this function may be applied to combine the s_j values into a "pooled" standard deviation s_{pool}.

Examples for such functions are

$$s_j = \text{const} \tag{7.1.1-7}$$

$$s_j^2 = s_0^2 + s_r^2 \left(L_{Tj} / Max\right)^2 \tag{7.1.1-8}$$

The components s_0^2 and s_r^2 have to be determined either by a graph or by calculation.

Note: For a standard deviation reported in a calibration certificate, it should be clear whether it is related to a single indication or to the mean of n indications.

7.1.1.4 δI_{ecc} accounts for the error due to off-centre position of the centre of gravity of a test load. This effect may occur where a test load is made up of more than one body. Where this effect cannot be neglected, an estimate of its magnitude may be based on these assumptions:

the differences determined by (6.3-1) are proportional to the distance of the load from the centre of the load receptor and to the value of the load;
the eccentricity of the effective centre of gravity of the test load is no more than $1/2$ of the value at the eccentricity test.

While there may be instruments on which the effect of eccentric loading is even greater at other angles than those where the test loads have been applied, based on the largest of the differences determined as per 6.3, the effect is estimated to be

$$\delta I_{ecc} \le \left\{ \left|\Delta I_{ecc,i}\right|_{\max} / \left(2 L_{ecc}\right) \right\} I \tag{7.1.1-9}$$

Rectangular distribution is assumed, so the standard uncertainty is

$$u\left(\delta I_{ecc}\right) = I \left|\Delta I_{ecc,i}\right|_{\max} / \left(2 L_{ecc} \sqrt{3}\right) \tag{7.1.1-10}$$

or, in relative notation,

$$\hat{w}\left(\delta I_{ecc}\right) = \left|\Delta I_{ecc,i}\right|_{\max} / \left(2 L_{ecc} \sqrt{3}\right) \tag{7.1.1-11}$$

7.1.1.5 The standard uncertainty of the indication is normally obtained by

$$u^2(I) = d_0^2/12 + d_I^2/12 + s^2(I) + \hat{w}^2\left(\delta I_{ecc}\right) I^2 \tag{7.1.1-12}$$

Note 1: the uncertainty $u(I)$ is = constant only where s = constant and no eccentricity error has to be considered.

Note 2: the first two terms on the right hand side may have to be modified in special cases as mentioned in 7.1.1.1 and 7.1.1.2.

7.1.2 Standard uncertainty of the reference mass

From 4.2.4 and 4.3.1 the reference value of mass is:

$$m_{ref} = m_N + \delta m_c + \delta m_B + \delta m_D + \delta m_{conv} + \delta m_{\dots} \tag{7.1.2-1}$$

The rightmost term stands for further corrections that may in special conditions be necessary to apply, it is not further considered hereafter.

The corrections and their standard uncertainties are:

7.1.2.1 δm_c is the correction to m_N to obtain the actual conventional value of mass m_c; given in the calibration certificate for the standard weights, together with the uncertainty of calibration U and the coverage factor k. The Standard uncertainty is

$$u(\delta m_c) = U/k \tag{7.1.2-2}$$

Where the Standard weight has been calibrated to specified tolerances Tol, e.g. to the mpe given in R 111, and where it is used its nominal value m_N, then δm_c = 0, and rectangular distribution is assumed, therefore

$$u(\delta m_c) = Tol/\sqrt{3} \tag{7.1.2-3}$$

Where a test load consists of more than one standard weight, the standard uncertainties are summed up arithmetically not by sum of squares, to account for assumed correlations

For test loads partially made up of substitution loads see 7.1.2.6

Note 1: cf. 6.2.1 for use of m_c or m_N.

Note 2: Where conformity of the standard weight(s) to R 111 is established, (7.1.2-3) may be modified by replacing Tol by *mpe*. For weights of m_N ≥ 0,1 kg the quotient *mpe*/ m_N is constant for all weights belonging to the same accuracy class, *mpe* = $c_{class} m_N$ with c_{class} from Table 7.1-1.

(7.1.2-3) may then be used in the form

$$u(\delta m_c) = m_N c_{class}/\sqrt{3} \tag{7.1.2-3a}$$

or as relative standard uncertainty

$$\hat{w}(\delta m_c) = c_{class}/\sqrt{3} \tag{7.1.2-3b}$$

Table 7.1-1 Quotient $c_{class} = mpe/m_N$ for standard weights m_N ≥ 100 g according to R 111 [4]

Class	c_{class} × 10^6
E_1	0,5
E_2	1,6
F_1	5
F_2	16
M_1	50
M_2	160
M_3	500

For weights of nominal value of 2 x 10n of the following classes: E2, F2 and M2, the value of class c × 106 should be substituted by 1,5, 15 and 150 respectively.

7.1.2.2 δm_B is the correction for air buoyancy as introduced in 4.2.4. The value depends on the density ρ of the calibration weight, on the assumed range of air density ρ_a, and on the adjustment of the instrument – cf cases A and B in 4.2.4.

Case A:

$$\delta m_B = -m_N(\rho_a - \rho_0)(1/\rho - 1/\rho_c) \tag{7.1.2-4}$$

with the relative standard uncertainty from

$$\hat{w}^2(m_B) = u^2(\rho_a)(1/\rho - 1/\rho_c)^2 + (\rho_a - \rho_0)^2 u^2(\rho)/\rho^4 \tag{7.1.2-5}$$

Case B1:

$$\delta m_B = -m_{cCal}\left[(\rho_a - \rho_0)(1/\rho - 1/\rho_c) + \delta\rho_{as}/\rho_c\right] \tag{7.1.2-6}$$

with the relative standard uncertainty from

$$\hat{w}^2(m_B) = u^2(\rho_a)(1/\rho - 1/\rho_c)^2 + (\rho_a - \rho_0)^2 u^2(\rho)/\rho^4 + u^2(\delta\rho_{as})/\rho_c^2 \tag{7.1.2-7}$$

Case B2:

$$\delta m_B = -\, m_N(\rho_a - \rho_0)/\rho \tag{7.1.2-8}$$

with the relative standard uncertainty from

$$\hat{w}^2(m_B) = u^2(\rho_a)/\rho^2 + (\rho_a - \rho_0)^2 u^2(\rho)/\rho^4 \tag{7.1.2-9}$$

As far as values for ρ, $u(\rho)$, ρ_a and $u(\rho_a)$, are known, these values should be used to determine $\hat{w}(m_B)$.

The density ρ and its standard uncertainty may in the absence of such information, be estimated according to the state of the art. Appendix E1 offers internationally recognized values for common materials used for standard weights.

The air density ρ_a and its standard uncertainty can be calculated from temperature and barometric pressure if available (the relative humidity being of minor influence), or may be estimated from the altitude above sea-level.

For the difference $\delta\rho_{as}$ (Case B1), zero may be assumed with an appropriate uncertainty $u(\delta\rho_{as})$ for which a limit $\Delta\rho_{as}$ should be estimated taking into account the variability of barometric pressure and temperature at the site, over a longer period of time.

A simple approach may be to use the same estimates for ρ_a and ρ_{as} the same uncertainty for both values.

Appendix A offers several formulae, and information about expected variances.

Appendix E offers values of $\hat{w}(m_B)$ for some selected combinations of values for ρ and ρ_a. For case A calibrations, the values are mostly negligible.

For case B calibrations, it may mostly be advisable to not apply a correction δm_B but to calculate the uncertainty based on ρ and on $\rho_a = \rho_0 \pm \Delta\rho_a$

Where conformity of the standard weights to R 111[4], is established, and no information on ρ and ρ_a, is at hand, recourse may be taken to section 10 of R 111[5]. No correction is applied, and the relative uncertainties are

for case A,

$$\hat{w}(m_B) \approx mpe / \left(4m_N \sqrt{3}\right) \tag{7.1.2-5a}$$

For cases B1 and B2,

$$\hat{w}(m_B) \approx \left(0{,}1\rho_0/\rho_c + mpe/(4m_N)\right)/\sqrt{3} \tag{7.1.2-9a}$$

From the requirement in footnote 5, these limits can be derived for ρ:
For class E_2: $|\rho - \rho_c| \leq$ 200 kg/m³, and for class F_1: $|\rho - \rho_c| \leq$ 600 kg/m³.

Note: Due to the fact that the density of materials used for standard weights is normally closer to ρ_c than the R111 limits would allow, the last 2 formulae may be considered as upper limits for $\hat{w}(m_B)$. Where a simple comparison of these values with the resolution of the instrument $(1/n_M = d/Max)$ shows they are small enough, a more elaborate calculation of this uncertainty component based on actual data, may be superfluous.

7.1.2.3 δm_D is a correction for a possible drift of m_c since the last calibration. A limiting value D is best assumed, based on the difference in m_c evident from consecutive calibration certificates of the standard weights.

[5] The density of the material used for weights shall be such that a deviation of 10 % from the specified air density (1.2 kg/m³) does not produce an error exceeding one quarter of the maximum permissible error.

In the absence of such information, D may be estimated in view of the quality of the weights, and frequency and care of their use, to a multiple of their expanded uncertainty $U(m_c)$:

$$D = k_D U(m_c) \tag{7.1.2-10}$$

where k_D may be chosen from 1 to 3.

It is not advised to apply a correction but to assume even distribution within $\pm D$ (rectangular distribution). The standard uncertainty is then

$$u(\delta m_D) = D/\sqrt{3} \tag{7.1.2-11}$$

Where a set of weights has been calibrated with a standardized expanded relative uncertainty $\hat{W}(m_c)$, it may be convenient to introduce a relative limit value for drift $D_{rel} = D/m_N$ and a relative uncertainty for drift

$$\hat{w}(m_D) = D_{rel}/\sqrt{3} = k_D \hat{W}(m_N)/\sqrt{3} \tag{7.1.2-12}$$

For weights conforming to R111 [4], the estimate may be $D \le mpe$, or $D_{rel} \le c_{class}$ – see Table 7.1-1

7.1.2.4 δm_{conv} is a correction for convection effects as per 4.2.3. A limiting value Δm_{conv} may be taken from Appendix F, depending on a known difference in temperature ΔT and on the mass of the standard weight.

It is not advised to apply a correction but to assume even distribution within $\pm \Delta m_{conv}$. The standard uncertainty is then

$$u(\delta m_{conv}) = \Delta m_{conv}/\sqrt{3} \tag{7.1.2-13}$$

7.1.2.5 The standard uncertainty of the reference mass is obtained from – cf. 7.1.2

$$u^2(m_{ref}) = u^2(\delta m_c) + u^2(\delta m_B) + u^2(\delta m_D) + u^2(\delta m_{conv}) \tag{7.1.2-14}$$

with the contributions from 7.1.2.1 to 7.1.2.4

As an example the terms are specified for a case A calibration with standard weights of $m_N \ge$ 0,1 kg conforming to R111, used with their nominal values:

$$\hat{w}^2(m_{ref}) = c_{class}^2/3 + c_{class}^2/48 + c_{class}^2/3 + (\Delta m_{conv}/m_N)^2/3 \tag{7.1.2-14a}$$

7.1.2.6 Where a test load is partially made up of substitution loads as per 4.3.3, the standard uncertainty for the sum $L_{Tn} = nm_{c1} + \Delta I_1 + \Delta I_2 + \ldots + \Delta I_{n-1}$ is given by the following expression:

$$u^2(L_{Tn}) = n^2 u^2(m_{c1}) + 2\left[u^2(I_1) + u^2(I_2) + \ldots + u^2(I_{n-1})\right] \tag{7.1.2-15}$$

with $u(m_{c1}) = u(m_{ref})$ from 7.1.2.5, and $u(I_j)$ from 7.1.1.5 for $I = I(L_{Tj})$

Note: the uncertainties $u(I_j)$ have to be included also for indications where the substitution load has been so adjusted that the corresponding ΔI becomes zero!

Depending on the kind of the substitution load, it may be necessary to add further uncertainty contributions:

for eccentric loading as per 7.1.1.4 to some or all of the actual indications $I(L_{Tj})$;

for air buoyancy of the substitution loads, where these are made up of low density materials (e.g. sand, gravel) and the air density varies significantly over the time the substitution loads are in use.

Where $u(I_j)$ = const, the expression simplifies to

$$u^2(L_{Tn}) = n^2 u^2(m_{c1}) + 2[(n-1)u^2(I)] \tag{7.1.2-16}$$

7.1.3 Standard uncertainty of the error

The standard uncertainty of the error is, with the terms from 7.1.1 and 7.1.2, as appropriate, calculated from

$$\begin{aligned} u^2(E) &= d_0^2/12 + d_I^2/12 + s^2(I) + u^2(\delta I_{ecc}) \\ &+ u^2(\delta m_c) + u^2(\delta m_B) + u^2(\delta m_D) + u^2(\delta m_{conv}) \end{aligned} \tag{7.1.3-1a}$$

or, where relative uncertainties apply, from

$$\begin{aligned} u^2(E) &= d_0^2/12 + d_I^2/12 + s^2(I) + \hat{w}^2(I_{ecc})I^2 \\ &+ \{\hat{w}^2(m_c) + \hat{w}^2(m_B) + \hat{w}^2(m_D)\}m_{ref}^2 + u^2(\delta m_{conv}) \end{aligned} \tag{7.1.3-1b}$$

All input quantities are considered to be uncorrelated, therefore covariances are not considered.

The index “ j ”has been omitted. Where the last terms in (7.1.3-1a, b) are small compared to the first 3 terms, the uncertainty of all errors determined over the weighing range is likely to be quite similar. If this is not the case, the uncertainty has to be calculated individually for each indication.

In view of the general experience that errors are normally very small compared to the indication, or may even be zero, in (7.1.3-1a, b) the values for m_{ref} and I may be replaced by I_N.

The terms in (7.1.3-1a, b) may then be grouped into a simple formula which better reflects the fact that some of the terms are absolute in nature while others are proportional to the indication:

$$u^2(E) = \alpha^2 + \beta^2 I^2 \tag{7.1.3-2}$$

Where (7.1.1-7) or (7.1.1-8) applies to the standard deviation determined for the calibrated instrument, the corresponding terms are of course included in (7.1.3-2).

7.2 *Standard uncertainty for a characteristic*

Where an approximation is performed to obtain a formula $E = f(I)$ for the whole weighing range as per 6.2.2, the standard uncertainty of the error per 7.1.3 has to be modified to be consistent with the method of approximation. Depending on the model function, this may be

a single variance $u^2{}_{appr}$ which is added to (7.1.3-1), or

a set of variances and covariances which include the variances in (7.1.3-1).

The calculations should also include a check whether the model function is mathematically consistent with the data sets E_j, I_j, $u(E_j)$.

The $\min \chi^2$, approach which is similar to the least squares approach, is proposed for approximations. Details are given in Appendix C.

7.3 *Expanded uncertainty at calibration*

The expanded uncertainty of the error is

$$U(E) = ku(E) \tag{7.3-1}$$

The coverage factor k, should be chosen such that the expanded uncertainty corresponds to a coverage probability of approximately 95 %.

The value $k = 2$, corresponding to a 95,5% probability, applies where

a) a normal (Gaussian) distribution can be attributed to the error of indication,

and

b) the standard uncertainty $u(E)$ is of sufficient reliability (i.e. it has a sufficient number of degrees of freedom).

Appendix B2 offers additional information to these conditions, and Appendix B3 advises how to determine the factor k where one or both of them are not met.

It is acceptable to determine only one value of k, for the "worst case" situation identified by experience, which may be applied to the standard uncertainties of all errors of the same weighing range.

7.4 *Standard uncertainty of a weighing result*

The user of an instrument should be aware of the fact that in normal usage of an instrument that has been calibrated, the situation is different from that at calibration in some if not all of these aspects:

1. the indications obtained for weighed bodies are not the ones at calibration,
2. the weighing process may be different from the procedure at calibration:
    a. certainly only one reading for each load, not several readings to obtain a mean value,
    b. reading to the scale interval d, of the instrument, not with higher resolution,

c. loading up and down, not only upwards – or vice versa,

d. load kept on load receptor for a longer time, not unloading after each loading step – or vice versa,

e. eccentric application of the load,

f. use of tare balancing device, etc.

3. the environment (temperature, barometric pressure etc.) may be different,
4. on instruments which are not readjusted regularly e.g. by use of a built-in device, the adjustment may have changed, due to ageing or to wear and tear.
   Unlike the items 1 to 3, this effect is usually depending on the time that has elapsed since the calibration, it should therefore be considered in relation to a certain period of time, e.g. for one year or the normal interval between calibrations.

In order to clearly distinguish from the indications I obtained during calibration, the weighing results obtained when weighing a load L on the calibrated instrument, these terms and symbols are introduced:

R = reading, any indication obtained after the calibration;
W = weighing result, reading corrected for the error E.

R is understood as a single reading in normal resolution (multiple of d), with corrections to be applied as applicable.

For a reading taken under the same conditions as those prevailing at calibration, for a load well centred on the load receptor, only corrections to account for points 2a and 2b above apply. The result may be denominated as the weighing result under the conditions of the calibration $W*$:

$$W* = R + \delta R_{digL} + \delta R_{rep} - \left(R_0 + \delta R_{dig0}\right) - E \qquad (7.4\text{-}1a)$$

with the associated uncertainty

$$u(W*) = \sqrt{\left\{u^2(E) + u^2\left(\delta R_{dig0}\right) + u^2\left(\delta R_{digL}\right) + u^2\left(\delta R_{rep}\right)\right\}} \qquad (7.4\text{-}2a)$$

$W*$ and $u(W*)$ can be determined directly using the information, and the results of the calibration as given in the calibration certificate:

Data sets I_{cal}, E_{cal}, $U(E_{cal})$, and/or
A characteristic $E(R) = f(I)$ and $U(E(R)) = g(I)$.

This is done in 7.4.1. and in 7.4.2.

To take account of the remaining possible influences on the weighing result, further corrections are formally added to the reading in a general manner resulting in the weighing result in general:

$$W = W* + \delta R_{instr} + \delta R_{proc} \qquad (7.4\text{-}1b)$$

with the associated uncertainty

$$u(W) = \sqrt{u^2(W^*) + u^2(\delta R_{instr}) + u^2(\delta R_{proc})} \qquad (7.4\text{-}2b)$$

The added terms and the corresponding standard uncertainties are discussed in 7.4.3 and 7.4.4. The standard uncertainties $u(W^*)$ and $u(W)$ are finally presented in 7.4.5.
Sections 7.4.3 and 7.4.4, and the information on $u(W)$ and $U(W)$ in sections 7.4.5 and 7.5, are meant as advice to the user of the instrument on how to estimate the uncertainty of weighing results obtained under his normal conditions of use. They are not meant to be exhaustive nor mandatory.

Where a calibration laboratory offers such estimates to its clients which are based upon information that has not been measured by the laboratory, the estimates may not be presented as part of the calibration certificate.

7.4.1 Standard uncertainty of a reading in use

To account for sources of variability of the reading, (7.1.1-1) applies, with I replaced by R:

$$R = R_L + \delta R_{digL} + \delta R_{rep} - (R_0 + \delta R_{dig0}) \cdots \{+ \delta R_{ecc}\} \qquad (7.4.1\text{-}1)$$

Term in $\{\ \}$ to be added if applicable

The corrections and their standard uncertainties are:

7.4.1.1 δR_{dig0} accounts for the rounding error at zero reading. 7.1.1.1 applies with the exception that the variant $d_T < d$, is excluded, so

$$u(\delta R_{dig0}) = d_0 / \sqrt{12} \qquad (7.4.1\text{-}2)$$

Note 2 in 7.1.1.1 applies.

7.4.1.2 δR_{digL} accounts for the rounding error at load reading. 7.1.1.2 applies with the exception that the variant $d_T < d_L$ is excluded, so

$$u(\delta R_{digL}) = d_L / \sqrt{12} \qquad (7.4.1\text{-}3)$$

7.4.1.3 δR_{rep} accounts for the error due to imperfect repeatability. 7.1.3.1 applies, the relevant standard deviation s or $s(I)$ for a single reading, is to be taken from the calibration certificate, so

$$u(\delta R_{rep}) = s \text{ or } u(\delta R_{rep}) = s(R) \qquad (7.4.1\text{-}4)$$

Note: In the calibration certificate, the standard deviation may be reported as being related to a single indication, or to the mean of *n* indications. In the latter case, the value of s has to be multiplied by $\sqrt{n}$ to give the standard deviation for a single reading.

7.4.1.4 δR_{ecc} accounts for the error due to off-centre position of the centre of gravity of a load. It has been put in brackets as it is normally relevant only for W not for W^*, and will therefore be considered in 7.4.4.3.

7.4.1.5 The standard uncertainty of the reading is then obtained by

$$u^2(R) = d_0^2/12 + d_R^2/12 + s^2(R) \cdots \left\{ + \hat{w}^2(R_{ecc})R^2 \right\} \qquad (7.4.1\text{-}5)$$

Term in $\{ \ \}$ to be added if applicable

Note: the uncertainty $u(R)$ is = constant where s = constant; where in exceptional cases the eccentricity error has to be considered, the term should be taken from 7.4.4.4.

7.4.2 Uncertainty of the error of a reading

Where a reading R corresponds to an indication I_{calj} reported in the calibration certificate, $u(E_{calj})$ may be taken from there. For other readings, $u(E(R))$ may be calculated by (7.1.3-2) if α and β are known, or it results from interpolation, or from an approximation formula as per 7.2.

The uncertainty $u(E(R))$ is normally not smaller than $u(E_{calj})$ for an indication I_j that is close to the actual reading R, unless it has been determined by an approximation formula.

Note: the calibration certificate normally presents $U_{95}(E_{cal})$ from which $u(E_{cal})$ is to be derived considering the coverage factor k stated in the certificate.

7.4.3 Uncertainty from environmental influences

The correction term δR_{instr} accounts for up to 3 effects which are discussed hereafter. They do normally not apply to instruments which are adjusted right before they are actually being used – cf 4.2.4, case A. For other instruments they should be considered as applicable. No corrections are actually being applied, the corresponding uncertainties are estimated, based on the user's knowledge of the properties of the instrument.

7.4.3.1 A term δR_{temp} accounts for a change in the characteristic (or adjustment) of the instrument caused by a change in ambient temperature. A limiting value can be estimated to $\delta R_{temp} = TK\Delta T$ with the following terms.

Normally there is a manufacturer's specification like $TK = \partial(Max)/\partial T$, in many cases quoted as $|TK| \leq |TC|$ in 10^{-6}/K. By default, for instruments with type approval under OIML 76 [3], it may be assumed $|TC| \leq mpe(Max)/(Max\Delta T_{Appr})$ where ΔT_{Appr} is the temperature range of approval marked on the instrument; for other instruments, either a conservative assumption has to be made, leading to a multiple (3 to 10 times) of the comparable value for instruments with type approval, or no information can be given at all for a use of the instrument at other temperatures than that at calibration.

The range of variation of temperature ΔT (full width) should be estimated in view of the site where the instrument is being used, as discussed in Appendix A.2.2.

Rectangular distribution is assumed, therefore the relative uncertainty is

$$\hat{w}(R_{temp}) = TC\Delta T / \sqrt{12} \quad (7.4.3\text{-}1)$$

7.4.3.2 A term δR_{bouy} accounts for a change in the adjustment of the instrument due to the variation of the air density; no correction to be applied, uncertainty contribution to be considered as in 7.1.2.2, where a variability of the air density larger than that at calibration is expected.

Note: the density ρ of the weighed body is not considered in this uncertainty contribution, as it is constitutive for the value of the weighing result W !.

7.4.3.3 A term δR_{adj} accounts for a change in the adjustment of the instrument since the time of calibration due to ageing, or wear and tear.

A limiting value may be taken from previous calibrations where they exist, as the largest difference $|\Delta E(Max)|$ in the errors at or near *Max* between any two consecutive calibrations. By default, $\Delta E(Max)$ should be taken from the manufacturer's specification for the instrument, or may be estimated to $\Delta E(Max) = mpe(Max)$ for instruments conforming to a type approval under OIML R76 [2] (or EN 45501 [3]). Any such value can be considered in view of the expected time interval between calibrations, assuming fairly linear progress of the change with time.
Rectangular distribution is assumed, therefore the relative uncertainty is

$$\hat{w}(R_{adj}) = |\Delta E(Max)| / \left(Max\sqrt{3}\right) \quad (7.4.3\text{-}2)$$

7.4.3.4 The relative standard uncertainty related to errors resulting from environmental effects is calculated by

$$\hat{w}^2(R_{instr}) = \hat{w}^2(R_{temp}) + \hat{w}^2(R_{adj}) \quad (7.4.3\text{-}3)$$

7.4.4 Uncertainty from the operation of the instrument

The correction term δR_{proc} accounts for additional errors which may occur where the weighing procedure is different from the one(s) at calibration. No corrections are actually being applied but the corresponding uncertainties are estimated, based on the user's knowledge of the properties of the instrument.

7.4.4.1 A term δR_{Tare} accounts for a Net weighing result after a tare balancing operation [3]. The possible error and the uncertainty assigned to it should be estimated considering the basic relation between the readings involved:

$$R_{Net} = R'_{Gross} - R'_{Tare} \quad (7.4.4\text{-}1)$$

where the R' are fictitious readings which are processed inside the instrument, while the visible indication R'_{Net} is obtained directly, after setting the instrument's indication to zero with the tare load on the load receptor. The weighing result in this case is in theory:

$$W_{Net} = R_{Net} - [E_{cal}(Gross) - E_{cal}(Tare)] - \delta R_{instr} - \delta R_{proc} \quad (7.4.4\text{-}2)$$

consistent with (7.3-1). The errors at Gross and Tare would have to be taken as errors for equivalent R values as above. However, the Tare value – and consequently the Gross value – are not normally recorded.

The error may then be estimated to

$$E_{Net} = E(Net) + \delta R_{Tare} \tag{7.4.4-3}$$

where $E(Net)$ = is the error for a reading R = Net with an additional correction for the effect of non-linearity of the error curve $E_{cal}(I)$. To quantify the non-linearity, recourse may be taken to the first derivative of the function $E_{appr} = f(R)$, is known, or the slope q_E between consecutive calibration points may be calculated by

$$q_E = \frac{\Delta E_{cal}}{\Delta I} = \frac{E_{j+1} - E_j}{I_{j+1} - I_j} \tag{7.4.4-4}$$

The largest and the smallest value of the derivatives or of the quotients are taken as limiting values for the correction $\delta R_{Tare} = q_E R_{Net}$, for which rectangular distribution may be assumed. This results in the relative standard uncertainty

$$\hat{w}(R_{Tare}) = (q_{E\max} - q_{E\min}) / \sqrt{12} \tag{7.4.4-5}$$

To estimate the uncertainty $u(W)$, (7.1-2) is applied with $R = R_{Net}$. For $u(E)$ it is justified to assume $u(E(Net)) = u(E_{cal}(R = Net))$ because of the full correlation between the quantities contributing to the uncertainties of the errors of the fictitious *Gross* and *Tare* readings.

7.4.4.2 A term δR_{time} accounts for possible effects of creep and hysteresis, in situations like the following:

a) loading at calibration was continuously upwards, or continuously up-and downwards (method 2 or 3 in 5.2), so the load remained on the load receptor for a certain period of time; this is quite significant where the substitution method has been applied, usually on high capacity instruments. When in normal use, a discrete load to be weighed is put on the load receptor and is kept there just as long as is necessary to obtain a reading or a printout, the error of indication may differ from the value obtained for the same load at calibration.

Where tests were performed continuously up and down, the largest difference of errors ΔE_j for any test load m_j may be taken as the limiting value for this effect, leading to a relative standard uncertainty

$$\hat{w}(R_{time}) = \Delta E_{j\max} / \left(m_j \sqrt{12}\right) \tag{7.4.4-6}$$

Where tests were performed only upwards, the error on return to Zero E_0, if determined may be used to estimate a relative standard uncertainty

$$\hat{w}(R_{time}) = E_0 / \left(Max\sqrt{3}\right) \tag{7.4.4-7}$$

In the absence of such information, the limiting value may be estimated for instruments with type approval under OIML R76 [2] (or EN 45501 [3]). Any as

$$\Delta E(R) = Rmpe(Max)/Max \tag{7.4.4-8}$$

For instruments without such type approval, a conservative estimate would be a multiple (n = 3 to 10 times) of this value.

The relative standard uncertainty is

$$\hat{w}(R_{time}) = mpe(Max)/\left(Max\sqrt{3}\right), \text{ or } = nmpe(Max)/\left(Max\sqrt{3}\right) \tag{7.4.4-9}$$

b) loading at calibration was with unloading between load steps, loads to be weighed are kept on the load receptor for a longer period. In the absence of any other information – e.g. observation of the change in indication over a typical period of time – recourse may be taken to (7.4.4-9) as applicable.

c) loading at calibration was only upwards, discharge weighing is performed in use. This situation may be treated as the inverse of the tare balancing operation – see 7.4.4.1 - combined with point b) above. (7.4.4-5) and (7.4.4-9) apply.

Note: In case of discharge weighing, the reading R shall be taken as a positive value although it may be indicated as negative by the weighing instrument.

7.4.4.3 δR_{ecc} accounts for the error due to off-centre position of the centre of gravity of a load. 7.1.1.4 applies with the modification that the effect found during calibration should be considered in full, so

$$w(R_{ecc}) = \left(\Delta I_{ecc,i}\right)_{\max} / \left(L_{ecc}\sqrt{3}\right) \tag{7.4.4-10}$$

7.4.4.4 where dynamic objects e.g. live animals are weighed, it is assumed that $u(\delta I_{rep})$ will be increased. A typical object should therefore be used to determine the standard deviation s_{dyn} of at least 5 weighings, and $s(R)$ in (7.4.1-5) should be replaced by s_{dyn}.

7.4.5 Standard uncertainty of a weighing result

The standard uncertainty of a weighing result is calculated from the terms specified in 7.4.1 to 7.4.4, as applicable.

For the weighing result under the conditions of the calibration:

$$u^2(W^*) = d_0^2/12 + d_L^2/12 + s^2(R) + u^2(E) \tag{7.4.5-1a}$$

For the weighing result in general:

$$u^2(W) = \boldsymbol{u^2(W^*)} + \left[\hat{w}^2(R_{temp}) + \hat{w}^2(R_{adj}) + \hat{w}^2(R_{Tare}) + \hat{w}^2(R_{time}) + \hat{w}^2(R_{ecc})\right]R^2 + \left[s_{dyn}^2 - s^2(R)\right] \tag{7.4.5-1b}$$

The term $\boldsymbol{u^2(W*)}$ has been put in bold to indicate that it applies in any case, whereas the other terms should be included as applicable.

The many contributions to $u(W)$ may be grouped in two terms α_W^2 and β_W^2

$$u^2(W) = \alpha_W^2 + \beta_W^2 R^2 \tag{7.4.5-2}$$

where α_W^2 is the sum of squares of all absolute standard uncertainties, and β_W^2 = is the sum of squares of all relative standard uncertainties.

7.5 Expanded uncertainty of a weighing result

7.5.1 Errors accounted for by correction

The complete formula for a weighing result which is equal to the reading corrected for the error determined by calibration, is

$$W* = R - E(R) \pm U(W*) \tag{7.5.1a}$$

or

$$W = R - E(R) \pm U(W) \tag{7.5.1b}$$

as applicable.

The expanded uncertainty $U(W)$ is to be determined as

$$U(W*) = ku(W*) \tag{7.5-2a}$$

or

$$U(W) = ku(W) \tag{7.5-2a}$$

with $u(W*)$ or $u(W)$ as applicable from 7.4.5.

For $U(W*)$ the coverage factor k should be determined as per 7.3.
For $U(W)$ the coverage factor k will, due to the large number of terms constituting $u(W)$, in most cases k = 2 even where the standard deviation s is obtained from only few measurements, and/or where $k_{cal} > 2$ was stated in the calibration certificate.
In cases of doubt, k should be determined as per 7.3 for

$u(W(R = 0)) = \alpha_W$, and for

$$u(W(R = Max)) = \sqrt{\alpha_W^2 + \beta_W^2 Max^2}$$

7.5.2 Errors included in uncertainty

It may have been agreed by the calibration laboratory and the client to derive a "global uncertainty" $U_{gl}(W)$ which includes the errors of indication such that no corrections have to be applied to the readings in use:

$$W = R \pm U_{gl}(W) \tag{7.5.2-1}$$

Unless the errors are more or less centred about Zero, they form a one-sided contribution to the uncertainty which can only be treated in an approximate manner. For the sake of simplicity and convenience, the "global uncertainty" is best stated in the format of an expression for the whole weighing range, instead of individual values stated for fixed values of the weighing result.

Let $E(R)$ be a function, or E^0 be one value representative for all errors stated over the weighing range in the calibration certificate, then the combination with the uncertainties in use may in principle take on one of these forms:

$$U_{gl}(W) = k\sqrt{u^2(W) + (E(R))^2} \tag{7.5.2-2}$$

$$U_{gl}(W) = k\sqrt{u^2(W) + (E^0)^2} \tag{7.5.2-2a}$$

$$U_{gl}(W) = k\sqrt{u^2(W) + (E^0)^2 \left(R/Max\right)^2} \tag{7.5.2-2b}$$

$$U_{gl}(W) = ku(W) + |E(R)| \tag{7.5.2-3}$$

$$U_{gl}(W) = ku(W) + |E^0| \tag{7.5.2-3a}$$

$$U_{gl}(W) = ku(W) + |E^0| R/Max \tag{7.5.2-3b}$$

Considering the format for *u(W)* in (7.4.5-2b), the formulae (7.5.2-2b) and (7.5.2-3b) may be more suitable than the corresponding versions with letter "a".

For the generation of the formulae $E(R)$ or the representative value E^0 see Appendix C.

It is important to ensure that $U_{gl}(W)$ retains a coverage probability of not less than 95 % over the whole weighing range.

7.5.3 Other ways of qualification of the instrument

A client may expect from, or have asked the Calibration Laboratory for a statement of conformity to a given specification, as $|W - R| \leq Tol$ with Tol being the applicable tolerance. The tolerance may be specified as "$Tol = x\%$ of R", as "$Tol = nd$", or the like.

Conformity may be declared, in consistency with ISO/IEC 17025 under condition that

$$|E(R) + U(W(R))| \leq Tol(R) \tag{7.5.3-1}$$

either for individual values of R or for any values within the whole or part of the weighing range.

Within the same weighing range, conformity may be declared for different parts of the weighing range, to different values of *Tol*.

8 CALIBRATION CERTIFICATE

This section contains advice on what information may be useful to be given in a calibration certificate. It is intended to be consistent with the requirements of ISO/IEC 17025 which take precedence.

8.1 General Information

Identification of the Calibration Laboratory,
reference to the accreditation (accrediting body, number of the accreditation),
identification of the certificate (calibration number, date of issue, number of pages),
signature(s) of authorised person(s).

Identification of the client.

Identification of the calibrated instrument,
information about the instrument (Manufacturer, kind of instrument, Max, d, place of installation).

Warning that the certificate may be reproduced only in full unless the calibration laboratory permits otherwise in writing.

8.2 Information about the calibration procedure

Date of measurements,
site of calibration, and place of installation if different,
conditions of environment and/or use that may affect the results of the calibration.

Information about the instrument (adjustment performed, any anomalies of functions, setting of software as far as relevant for the calibration, etc.).

Reference to, or description of the applied procedure, as far as this is not obvious from the certificate, e.g. constant time interval observed between loadings and/or readings.

Agreements with the client e.g. over limited range of calibration, metrological specifications to which conformity is declared.

Information about the traceability of the measuring results.

8.3 Results of measurement

Indications and/or errors for applied test loads, or errors related to indications – as discrete values and/or by an equation resulting from approximation,

details of the loading procedure if relevant for the understanding of the above,

standard deviation(s) determined, identified as related to a single indication or to the mean of indications,

expanded uncertainty of measurement for the reported results.

Indication of the expansion factor k, with comment on coverage probability, and reason for $k \neq 2$ where applicable.

Where the indications/errors have not been determined by normal readings - single readings with the normal resolution of the instrument - , a warning should be given that the reported uncertainty is smaller than what would be found with normal readings.

For clients that are less knowledgeable, advice might be useful where applicable, on:

the definition of the error of indication,
how to correct readings in use by subtracting the corresponding errors,
how to interpret indications and/or errors presented with smaller decimals

than the scale interval d.

It may be useful to quote the values of $U(W^*)$ for either all individual errors or for the function $E(R)$ resulting from approximation.

8.4 Additional information

Additional information about the uncertainty of measurement expected in use, inclusive of conditions under which it is applicable, may be attached to the certificate without becoming a part of it.

Where errors are to be accounted for by correction, this formula could be used:

$$W = R - E(R) \pm U(W) \tag{8.4-1}$$

accompanied by the equation for $E(R)$.

Where errors are included in the "global uncertainty", this formula could be used:

$$W = R \pm U_{gl}(W) \tag{8.4-2}$$

A statement should be added that the expanded uncertainty of values from the formula corresponds to a coverage probability of at least 95 %.

Optional:
Statement of conformity to a given specification, and range of validity where applicable.

This statement may take the form

$$W = R \pm Tol \tag{8.4-3}$$

and may be given

in addition to the results of measurement, or

as stand-alone statement, with reference to the results of measurement declared to be retained at the calibration Laboratory.

The statement may be accompanied by a comment indicating that all measurement results enlarged by the expanded uncertainty of measurement, are within the specification limits.

9 VALUE OF MASS OR CONVENTIONAL VALUE OF MASS

The quantity W is an estimate of the conventional value of mass m_c of the object weighed[6]. For certain applications it may be necessary to derive from W the value of mass m, or a more accurate value for m_c.

The density ρ or the volume V of the object, together with an estimate of their standard uncertainty, must be known from other sources.

[6] In the majority of cases, especially when the results are used for trade, the value W is used as the result of the weighing

9.1 Value of mass

The mass of the object is

$$m = W\left[1 + \rho_a \left(1/\rho - 1/\rho_c\right)\right] \tag{9.1-1}$$

Neglecting terms of second and higher order, the relative standard uncertainty $\hat{w}(m)$ is given by

$$\hat{w}^2(m) = \frac{u^2(W)}{W^2} + u^2(\rho_a)\left(\frac{1}{\rho} - \frac{1}{\rho_c}\right)^2 + \rho_a^2 \frac{u^2(\rho)}{\rho^4} \tag{9.1-2}$$

For ρ_a and $u(\rho_a)$ (density of air) see Appendix A.

If V and $u(V)$ are known instead of ρ and $u(\rho)$, ρ may be approximated by W/V, and $\hat{w}(\rho)$ may be replaced by $\hat{w}(V)$.

9.2 Conventional value of mass

The conventional value of mass of the object is

$$m_c = W\left[1 + \left(\rho_a - \rho_0\right)\left(1/\rho - 1/\rho_c\right)\right] \tag{9.2-1}$$

Neglecting terms of second and higher order, the relative standard uncertainty $\hat{w}(m_c)$ is given by

$$\hat{w}^2(m_c) = \frac{u^2(W)}{W^2} + u^2(\rho_a)\left(\frac{1}{\rho} - \frac{1}{\rho_c}\right)^2 + \left(\rho_a - \rho_0\right)^2 \frac{u^2(\rho)}{\rho^4} \tag{9.2-2}$$

The same comments as given to (9.1-2) apply.

10 REFERENCES

[1] *Guide to the expression of Uncertainty in Measurement*, first edition 1993, ISO (Geneva, Switzerland)

[2] *OIML R 76: Non-automatic Weighing Instruments Part 1: Metrological Requirements - Tests,* edition 2006 (E)

[3] *EN 45501: Metrological Aspects of Non-automatic Weighing Instruments, edition 1992 with supplement 1994*

[4] *OIML R111, Weights of Classes E1, E2, F1, F2, M1, M1-2, M2, M2-3, M3,* Edition 2004 (E)

[5] *A Picard, R S Davis, M Gläser, K Fujii: Revised formula for the density of moist air (CIPM-2007).* Metrologia 45 (2008), p. 149-155.

[6] *M. Gläser: Change of the apparent mass of weights arising from temperature differences* Metrologia 36 (1999), p. 183-197

[7] *ISO 31 Quantities and Units (1993) Part 11: Mathematical Signs and Symbols for use in physical sciences and technology*

[8] *VIM, International Vocabulary of Basic and General Terms in Metrology,* 2nd edition 1994

[9] *Determination of Mass – Part 1: Dissemination of the unit of mass", by R. Balhorn, D. Buer, M. Gläser and M. Kochsiek* PTB-Bericht MA-24, 2nd revised edition, Braunschweig, April 1992

APPENDIX A: ADVICE FOR ESTIMATION OF AIR DENSITY

Note: In Appendix A, the symbols are *T* for temperature in *K*, and *t* for temperature in °C

A1 Formulae for the density of air

The most accurate formula to determine the density of moist air is the one recommended by CIPM [5][7]. For the purposes of this Guideline, less sophisticated formulae are sufficient which render slightly less precise results.

A1.1 Simplified version of CIPM-formula, exponential version

from [4], section E3

$$\rho_a = \frac{0{,}34848\,p - 0{,}009 h_r \exp(0{,}061 t)}{273{,}15 + t} \tag{A1.1-1}$$

with

ρ_a	air density in kg/m³
p	barometric pressure in hPa
h_r	relative humidity of air in %
t	air temperature in °C

Formula yields results with $u_{form} / \rho_a \leq$ 2,4×10⁻⁴ under the following conditions of environment (uncertainties of measurement of p, h_r, t not included)

600 hPa ≤ p ≤ 1 100 hPa

20 % ≤ h_r ≤ 80 %

15 °C ≤ t ≤ 27 °C

A1.2 Simplified version of CIPM-formula, standard version

From [9] this expression can be quoted

$$\rho_a = \frac{0{,}348444\,p - h_r(0{,}00252 t - 0{,}020582)}{273{,}15 + t} \tag{A1.2-1}$$

with symbols as above.

Formula yields results with $\Delta\rho_{a,form} \leq$ 0,001 41 kg/m³ under the following conditions of environment (uncertainties of measurement of p, h_r, t not included):

p	600 hPa ≤ p ≤ 1 100 hPa
h_r	20 % ≤ h_r ≤ 80 %
t	15 °C ≤ t ≤ 27 °C

[7] The recommended ranges of temperature and pressure over which the CIPM-2007 equation may be used are
600 hPa ≤ p ≤1 100 hPa
15 ºC ≤ t ≤ 27 ºC.

$\Delta\rho_{a,\,form}$ is the difference between values from this formula and the corresponding values from the CIPM formula. Therefore, the combined relative formula standard uncertainty $\hat{w}\left(\rho_{a,\,form}\right)$ is given by

$$\hat{w}^2\left(\rho_{a,form}\right) = (2{,}2\times10^{-5})^2 + ((0{,}001\ 41\ \text{kg/m}^3)/(1{,}2\ \text{kg/m}^3))^2\ /3$$

$$= 4{,}61\times10^{-7} \qquad \text{(A1.2-2)}$$

$$\hat{w}\left(\rho_{a,\,form}\right) = 6{,}79\ \text{x}10^{-4} \qquad \text{(A1.2-3)}$$

A1.3 Boyle-Mariotte- Formula

From the basic formula $p/\rho = RT$ it follows

$$\rho_a = \frac{\rho_{a,ref}T_{ref}\,p}{Tp_{ref}} \qquad \text{(A1.3-1)}$$

The reference values may be chosen at convenience. It could be the actual values determined at the time of calibration, or any other convenient set of values.

A very convenient modification to this formula can be offered as follows:

$$\rho_a = 0{,}992\,65\frac{\left(1{,}201\,31\ \text{kg/m}^3\right)\left(293{,}15\ \text{K}\right)p}{\left(273{,}15+t\right)\left(1\,015\ \text{hPa}\right)} \qquad \text{(A1.3-2)}$$

which yields values within ± 1,1% of the CIPM values - see A1.4 for justification, and range of validity.

A1.4 Formula errors

Sample calculations using EXCEL spreadsheets were performed to compare the results of air density obtained by the formulae above, against the CIPM values based on $x_{CO-2} = 0{,}000\,4$.

Comparisons were performed over the following steps of parameters:

Temperature	t	= 15 °C to 27 ºC
Barometric pressure	p	= 600 hPa to 1 100 hPa
Relative humidity	h_r	= 20 % to 80 %

The largest difference between any value from a simpler formula and the corresponding CIPM value, expressed in % of the CIPM value, was

Formula	Maximum Difference (absolute)
(A.1.1-1)	0,024 %
(A.1.2-1)	0,20 %
(A.1.3-1), Reference ρ_a = 1,200 21 kg/m³	1,4 %
(A.1.3-2), Reference ρ_a = 1,201 39 kg/m³	0,9 %

Note:

For ρ_a = 1,200 21 kg/m³, the reference values were

t = 20°C, p = 1 014 hPa, and h_r = 50%.

In the last line, the reference value were t = 20 °C, p = 1 015 hPa, h_r = 50 % while the reference air density has arbitrarily been set to

$\rho_{a,ref}$ = (1,201 39 kg/m³ x 0,994 62) to obtain a better fit relative to CIPM 2007.

A1.5 Average air density

Where measurement of temperature and barometric pressure is not possible, the mean air density at the site can be calculated from the altitude above sea level, as recommended in [4]:

$$\rho_a = \rho_0 \exp\left(-\frac{\rho_0}{p_0} gh\right) \tag{A1.5-1}$$

with p_0 = 101325 Pa

ρ_0 = 1,200 kg/m³

g = 9,81 m/s²

h = altitude above sea level in m

A2 Variations of parameters constituting the air density

A2.1 Barometric pressure:

The average barometric pressure p_{av} may be estimated form the altitude h in m above sea level SL of the location, using the relation

$$p(h) = p(SL) - h \times (0{,}12\ \mathrm{hPa/m}) \tag{A2.1-1}$$

with $p(SL)$ = 1 013,12 hPa

At any given location, the variations at most Δp = ± 40 hPa about the average[8]. Within these limits, the distribution is not rectangular as extreme values do occur only once in several years. It is more realistic to assume a normal distribution, with Δp being the "2σ" or even the "3σ". Hence

$u(\Delta p)$ = 20 hPa (for k = 2) or $u(\Delta p)$ = 13,3 hPa (for k = 3) (A2.1-2)

[8] Example: at Hannover, Germany, the difference between highest and lowest barometric pressures ever observed over 20 years was 77,1 hPa (Information from DWD, the German Meteorological Service) [1]

A2.2 Temperature

The possible variation $\Delta t = t_{\max} - t_{\min}$ of the temperature at the place of use of the instrument may be estimated from information which is easy to obtain:

limits stated by the client from his experience,
reading from suitable recording means,
setting of the control instrument, where the room is climatized or temperature stabilized;

in case of default sound judgement should be applied, leading to –e.g.

17 °C ≤ t ≤ 27 °C for closed office or laboratory rooms with windows,
Δt ≤ 5 K for closed rooms without windows in the centre of a building,
- 10 °C ≤ t ≤ + 30 °C or ≤ + 40 °C for open workshops, factory halls.

As has been said for the barometric pressure, a rectangular distribution is unlikely to occur for open workshops or factory halls where the atmospheric temperature prevails. However, to avoid different assumptions for different room situations, the assumption of rectangular distribution is recommended, leading to

$$u(\Delta t) = \Delta t / \sqrt{12} \qquad \text{(A2.2-1)}$$

A2.3. Relative humidity

The possible variation $\Delta h_r = h_{r,\max} - h_{r,\min}$ of the relative humidity at the place of use of the instrument may be estimated from information which is easy to obtain:

limits stated by the client from his experience,
reading from suitable recording means,
setting of the control instrument, where the room is climatized;

in case of default sound judgement should be applied, leading to – e.g.

30 % ≤ h_r ≤ 80 % for closed office or laboratory rooms with windows,
Δh_r ≤ 30 % or closed rooms without windows in the centre of a building,
20 % ≤ h_r ≤ 80 % for open workshops, factory halls.

It should be kept in mind that
at h_r < 40 % electrostatic effects may already influence the weighing result on high resolution instruments,
at h_r > 60 % corrosion may begin to occur.

As has been said for the barometric pressure, a rectangular distribution is unlikely to occur for open workshops or factory halls where the atmospheric relative humidity prevails. However, to avoid different assumptions for different room situations, the assumption of rectangular distribution is recommended, leading to

$$u(\Delta h_r) = \Delta h_r / \sqrt{12} \qquad \text{(A2.3-1)}$$

A3 Uncertainty of air density

The relative uncertainty of the air density $u(\rho_a)/\rho_a$ may be calculated by

$$u(\rho_a)/\rho_a = \sqrt{(u_p(\rho_a)/\rho_a \cdot u(p))^2 + (u_t(\rho_a)/\rho_a \cdot u(t))^2 + (u_{hr}(\rho_a)/\rho_a \cdot u(h_r))^2 + (u_{form}(\rho_a)/\rho_a)^2} \tag{A3.1-1}$$

with the sensitivity coefficients (derived from the CIPM formula for air density)

$u_p(\rho_a)/\rho_a$ = 1x10^{-3} hPa^{-1} for barometric pressure

$u_t(\rho_a)/\rho_a$ = - 4x10^{-3} °C^{-1} for air temperature

$u_{hr}(\rho_a)/\rho_a$ = -9x10^{-3} for relative humidity

Examples of standard uncertainty of air density, calculated for different parameters using the CIPM 2007 formula

Δp /hPa	Δt /°C	Δh_r	$\frac{u_p(\rho_a)}{\rho_a}u(p)$	$\frac{u_t(\rho_a)}{\rho_a}u(t)$	$\frac{u_{hr}(\rho_a)}{\rho_a}u(h_r)$	$\frac{u_{form}(\rho_a)}{\rho_a}$	$\frac{u(\rho_a)}{\rho_a}$
40	2	0,2	1,15 x 10^{-2}	-2,31 x 10^{-3}	-5,20 x 10^{-4}	2,20 x 10^{-5}	1,18 x 10^{-2}
40	2	1	1,15 x 10^{-2}	-2,31 x 10^{-3}	-2,60 x 10^{-3}	2,20 x 10^{-5}	1,21 x 10^{-2}
40	5	0,2	1,15 x 10^{-2}	-5,77 x 10^{-3}	-5,20 x 10^{-4}	2,20 x 10^{-5}	1,29 x 10^{-2}
40	5	1	1,15 x 10^{-2}	-5,77 x 10^{-3}	-2,60 x 10^{-3}	2,20 x 10^{-5}	1,32 x 10^{-2}
40	10	0,2	1,15 x 10^{-2}	-1,15 x 10^{-3}	-5,20 x 10^{-4}	2,20 x 10^{-5}	1,63 x 10^{-2}
40	10	1	1,15 x 10^{-2}	-1,15 x 10^{-3}	-2,60 x 10^{-3}	2,20 x 10^{-5}	1,65 x 10^{-2}
40	20	0,2	1,15 x 10^{-2}	-2,31 x 10^{-3}	-5,20 x 10^{-4}	2,20 x 10^{-5}	2,58 x 10^{-2}
40	20	1	1,15 x 10^{-2}	-2,31 x 10^{-3}	-2,60 x 10^{-3}	2,20 x 10^{-5}	2,60 x 10^{-2}
40	30	0,2	1,15 x 10^{-2}	-3,46 x 10^{-3}	-5,20 x 10^{-4}	2,20 x 10^{-5}	3,65 x 10^{-2}
40	30	1	1,15 x 10^{-2}	-3,46 x 10^{-3}	-2,60 x 10^{-3}	2,20 x 10^{-5}	3,66 x 10^{-2}
40	40	0,2	1,15 x 10^{-2}	-4,62 x 10^{-3}	-5,20 x 10^{-4}	2,20 x 10^{-5}	4,76 x 10^{-2}
40	40	1	1,15 x 10^{-2}	-4,62 x 10^{-3}	-2,60 x 10^{-3}	2,20 x 10^{-5}	4,77 x 10^{-2}
40	50	0,2	1,15 x 10^{-2}	-5,77 x 10^{-3}	-5,20 x 10^{-4}	2,20 x 10^{-5}	5,89 x 10^{-2}
40	50	1	1,15 x 10^{-2}	-5,77 x 10^{-3}	-2,60 x 10^{-3}	2,20 x 10^{-5}	5,89 x 10^{-2}

The relative uncertainty due to the formula of CIPM is 2,2 x 10-5 [5].
For approximations A.1.1-1, A.1.2-1 and A.1.3-1, the relative uncertainty due to the corresponding approximation formula should be properly substituted, $u_{form}(\rho_a)/\rho_a$.

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APPENDIX B: COVERAGE FACTOR k FOR EXPANDED UNCERTAINTY OF MEASUREMENT

Note: in this Appendix the general symbol *y* is used for the result of measurement, not a particular quantity as an indication, an error, a mass of a weighed body etc..

B1 Objective

The expansion factor *k* shall in all cases be chosen such that the expanded uncertainty of measurement has a coverage probability of approximately 95 %.

B2 Basic conditions for the application of k = 2

A factor *k* = 2 is to be applied where the following conditions are met:

normal distribution can be assigned to the output estimate *y* **and**
$u(y)$ **is sufficiently reliable**, see [1].

Normal distribution may be **assumed** where several (i.e.. *N* ≥ 3) uncertainty components, each derived from "well-behaved" distributions (normal, rectangular or the like), contribute to $u(y)$ by comparable amounts - see [1].

Note: this implies that none of the contributions with other than normal distribution, is of dominant value as defined in B.3.2.

Sufficient reliability is depending on the effective degrees of freedom. This criterion is met where no Type A contribution to $u(y)$ is based on less than 10 observations. see [1].

B3 Determining k in other cases

In any of the following cases, the expanded uncertainty is $U(y) = ku(y)$.

B3.1 Distribution assumed to be normal

Where the distribution of the output estimate y may be assumed to be normal, but $u(y)$ is not sufficiently reliable – see B.2 – then the effective degrees of freedom υ_{eff} have to be determined using the Welch-Satterthwaite formula, and *k* > 2 is read from the appropriate Table, as per [1], Appendix E.

B3.2 Distribution not normal

It may be obvious in a given situation that $u(y)$ contains one Type B uncertainty component $u_1(y)$ from a contribution whose distribution is not normal but e. g. rectangular or triangular, which is significantly greater than all the remaining components. In such a case, $u(y)$ is split up in the (possibly dominant) part u_1 and u_R = square root of $\sum u_j^2$ with j ≥ 2, the combined standard uncertainty comprising the remaining contributions, see [1].

If u_R ≤ 0,3 u_1, then u_1 is considered to be "dominant" and the distribution of y is considered to be essentially identical with that of the dominant contribution.

The expansion factor is chosen according to the shape of distribution of the dominant component:

for trapezoidal distribution with $\beta < 0{,}95$:

(β = edge parameter, ratio of smaller to larger edge of trapezoid)

$$k = \left\{1 - \sqrt{\left[0{,}05\left(1-\beta^2\right)\right]}\right\} / \sqrt{\left[\left(1+\beta^2\right)/6\right]}$$ - see [1]

for a rectangular distribution (β = 1): k = 1,65 – see [1]

for a triangular distribution (β = 0): k = 1,90

for U-type distribution: k = 1,41

The dominant component may itself be composed of 2 dominant components $u_1(y)$, $u_2(y)$, e.g. 2 rectangles making up one trapezoid, in which case u_R will be determined from the remaining u_j with $j \geq 3$

APPENDIX C: FORMULAE TO DESCRIBE ERRORS IN RELATION TO THE INDICATIONS

C1 Objective

This Appendix offers advice how to derive from the discrete values obtained at calibration and/or presented in a calibration certificate, errors and assigned uncertainties for any other reading R within the calibrated weighing range.

It is assumed that the calibration yields n sets of data I_{Nj}, E_j, U_j, or alternatively m_{Nj}, I_j, U_j, together with the expansion factor k and an indication of the distribution of E underlying k.

In any case, the nominal indication I_{Nj} is considered to be $I_{Nj} = m_{Nj}$.

It is further assumed that for any m_{Nj} the error E_j remains the same if I_j is replaced by I_{Nj}, it is therefore sufficient to look at the data I_{Nj}, E_j, u_j, and to omit the suffix N for simplicity's sake.

C2 Functional relations

C2.1 Interpolation

There are several polynomial formulae for interpolation[9], between values tabled versus equidistant arguments, which are rather easy to employ. The test loads may, however, in many cases not be equidistant which leads to quite complicated interpolation formulae if one looks for a single formula to cover the whole weighing range.

Linear interpolation between two adjacent points may be performed by

$$E(R) = E(I_k) + (R - I_k)(E_{k+1} - E_k)/(I_{k+1} - I_k) \tag{C2.1-1}$$

$$U(R) = U(I_k) + (R - I_k)(U_{k+1} - U_k)/(I_{k+1} - I_k) \tag{C2.1-2}$$

for a reading R with $I_k < R < I_{k+1}$. A higher order polynomial would be needed to estimate the possible interpolation error – this is not further elaborated.

C2.2 Approximation

Approximation should be performed by calculations or algorithms based on the "minimum χ^2" approach:

$$\chi^2 = \sum p_j v_j^2 = \sum p_j \left(f(I_j) - E_j\right)^2 = \text{minimum} \tag{C2.2-1}$$

with:

p_j = weighting factor (basically proportional to $1/u_j^2$)

v_j = residual

f = approximation function containing n_{par} parameters

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[9] An interpolation formula is understood to yield exactly the given values between which interpolation takes place. An approximation formula will normally not yield the given values exactly.

Together with the coefficients of the approximation function, the sum of the squares of the deviations should be determined as per (C.2.2-1), which is denominated by the term $\min\chi^2$. This serves to check the validity of the approximation.

If the following condition is met:

$$\left|\min\chi^2 - \upsilon\right| \leq \beta\sqrt{(2\upsilon)} \qquad \text{(C2.2-2)}$$

with

$\upsilon = n - n_{par}$ = degrees of freedom, and

$\beta =$ factor chosen to be 1, 2 (value applied most), or 3,

it is justified to assume the form of the model function $E(I)$ to be mathematically consistent with the data underlying the approximation.

C2.2.1 Approximation by polynomials

Approximation by a polynomial yields the general function

$$E(R) = f(R) = a_0 + a_1 R + a_2 R^2 + \ldots + a_{na} R^{na} \qquad \text{(C2.2-3)}$$

The suffix/exponent n_a of the coefficients should be chosen such that $n_{par} = n_a + 1 \leq n/2$.

The calculation is best performed by matrix calculation.

Let	$\mathbf{X}$	be a matrix whose n rows are (1, I_j, I_j^2,..., I_j^{na})
	$\mathbf{a}$	be a column vector whose components are the coefficients a_0, a_1, , a_{na} of the approximation polynomial
	$\mathbf{e}$	be a column vector whose n components are the E_j
	$\mathbf{U}(\mathbf{e})$	be the uncertainty matrix to the E_j.

$\mathbf{U}(\mathbf{e})$ is either a diagonal matrix whose elements are $u_{jj} = u^2(E_j)$, or it has been derived as a complete var/cov Matrix.

The weighting matrix $\mathbf{P}$ is

$$\mathbf{P} = \mathbf{U}(\mathbf{e})^{-1} \qquad \text{(C2.2-4)}$$

and the coefficients a_0, a_1,... are found by solving the normal equation

$$\mathbf{X^T P X a} - \mathbf{X^T P e} = 0 \qquad \text{(C2.2-5)}$$

with the solution

$$\mathbf{a} = \left(\mathbf{X^T P X}\right)^{-1} \mathbf{X^T P e} \qquad \text{(C2.2-6)}$$

The n deviations $v_j = f(I_j) - E_j$ are comprised in the vector

$$\mathbf{v} = \mathbf{Xa} - \mathbf{e} \tag{C2.2-7}$$

and $\min\chi^2$ is obtained by

$$\min\chi^2 = \mathbf{v}^\mathbf{T}\mathbf{Pv} \tag{C2.2-8}$$

Provided the condition of (C.2.2-2) is met, the variances and covariances for the coefficients a_i are given by the matrix

$$\mathbf{U}(\mathbf{a}) = \left(\mathbf{X}^\mathbf{T}\mathbf{PX}\right)^{-1} \tag{C2.2-9}$$

Where the condition of (C.2.2-2) is not met, one of these procedures may be applied:

a: repeat the approximation with a higher number n_a of coefficients, as long as $n_a + 1 \leq n/2$;

b: repeat the approximation after increasing all values u_j e.g. by multiplication with a suitable factor $c > 1$. ($\min\chi^2$ is proportional to $1/c^2$)

The results of the approximation, $\mathbf{a}$ and $\mathbf{U}(\mathbf{a})$ may be used to determine the approximated errors and the assigned uncertainties for the n points I_j.
The errors E_{apprj} j are comprised in the vector

$$\mathbf{e}_\mathbf{appr} = \mathbf{Xa} \tag{C2.2-10}$$

with the uncertainties given by

$$u^2\left(E_{apprj}\right) = \mathrm{diag}\left(\mathbf{XU}(\mathbf{a})\mathbf{X}^\mathbf{T}\right) \tag{C2.2-11}$$

They also serve to determine the error, and assigned uncertainty for any other indication – called a reading R to discriminate from the indications I_j – within the calibrated weighing range.

Let

$\mathbf{r}$ be a column vector whose elements are $\left(1, R, R^2, R^3, \ldots R^{na}\right)^T$,

$\mathbf{r}'$ be a column vector whose elements are the derivatives $\left(0, 1, 2R, 3R^2, \ldots n_a R^{na-1}\right)^T$

The error is

$$E_{appr}(R) = \mathbf{r}^\mathbf{T}\mathbf{a} \tag{C2.2-12}$$

And the uncertainty is given by

$$u^2(E_{appr}) = (\mathbf{r}'^{\mathrm{T}}\mathbf{a})\mathbf{U}(R)(\mathbf{r}'^{\mathrm{T}}\mathbf{a})^{\mathrm{T}} + \mathbf{r}^{\mathrm{T}}\mathbf{U}(\mathbf{a})\mathbf{r} \quad \text{(C2.2-13)}$$

The first term on the right hand side simplifies, as all 3 matrices are only onedimensional, to

$$(\mathbf{r}'^{\mathrm{T}}\mathbf{a})\mathbf{U}(R)(\mathbf{r}'^{\mathrm{T}}\mathbf{a})^{\mathrm{T}} = (a_1 + 2a_2R + 3a_3R^2 + ... + n_a a_{na} R^{na-1})^2 u^2(R) \quad \text{(C2.2-14)}$$

with $u^2(R) = d_0^2/12 + d_R^2/12 + s^2(I)$ as per (7.1.1.-11).

C2.2.2 Approximation by a straight line

Many modern electronic instruments are well designed, and corrected internally to achieve good linearity of the function $I = f(m)$. Therefore the errors are mostly resulting from incorrect adjustment, and are by and large increasing in proportion to R. For such instruments it may be quite appropriate to restrict the polynomial to a linear function, provided it is sufficient in view of the condition in (C.2.2-2).

The standard solution is to apply (C.2.2-3) with $n_a = 1$:

$$E(R) = f(R) = a_0 + a_1R \quad \text{(C2.2-15)}$$

One variant to this is to set $a_0 = 0$ and to determine only a_1. This can be justified by the fact that due to zero-setting – at least for increasing loads - the error $E(R = 0)$ is automatically = 0:

$$E(R) = f(R) = a_1R \quad \text{(C2.2-16)}$$

Another variant is to define the coefficient a $(= a_1$ in (C2.2-16)) as the mean of all gradients $q_j = E_j/I_j$. This allows inclusion of errors of net indications after a tare balancing operation if these have been determined at calibration:

$$a = \sum (E_j/I_j)/n \quad \text{(C2.2-17)}$$

The calculations, except for the variant (C2.2-17), may be performed using the matrix formulae in C.2.2.1.

Other possibilities are given hereafter.

C2.2.2.1 Linear regression as per (C2.2-12) may be performed by many standard pocket calculators.

Correspondence between results is typically,

"intercept"	⇔	a_0
"slope"	⇔	a_1

However, calculators may not be able to perform linear regression based on weighted error data, or linear regression with $a_0 = 0$.

C2.2.2.2 To facilitate programming the calculations by computer in non-matricial notation, the relevant formulae are presented hereafter. All formulae include the weighting factors $p_j = 1/u^2(E_j)$

For simplicity's sake, all indices " j " have been omitted from I , E , p

a) linear regression for (C2.2-15)

$$a_0 = \frac{\sum pE \sum pI^2 - \sum pI \sum pIE}{\sum p \sum pI^2 - \left(\sum pI\right)^2} \quad \text{(C2.2-15a)}$$

$$a_1 = \frac{\sum p \sum pIE - \sum pE \sum pI}{\sum p \sum pI^2 - \left(\sum pI\right)^2} \quad \text{(C2.2-15b)}$$

$$\min \chi^2 = \sum p\left(a_0 + a_1 I - E\right)^2 \quad \text{(C2.2-15c)}$$

$$u^2(a_0) = \frac{\sum pI^2}{\sum p \sum pI^2 - \left(\sum pI\right)^2} \quad \text{(C2.2-15d)}$$

$$u^2(a_1) = \frac{\sum p}{\sum p \sum pI^2 - \left(\sum pI\right)^2} \quad \text{(C2.2-15e)}$$

$$\operatorname{cov}(a_0, a_1) = \frac{\sum pI}{\sum p \sum pI^2 - \left(\sum pI\right)^2} \quad \text{(C2.2-15f)}$$

(C.2.2-15) applies fro the approximated error of the reading R ,and the uncertainty of the approximation $u\left(E_{appr}\right)$ is given by

$$u^2\left(E_{appr}\right) = a_1^2 u^2(R) + u^2(a_0) + R^2 u^2(a_1) + 2R\operatorname{cov}(a_0, a_1) \quad \text{(C2.2-15g)}$$

b) linear regression with a_0 = 0

$$a_1 = \sum pIE \Big/ \sum pI^2 \quad \text{(C2.2-16a)}$$

$$\min \chi^2 = \sum p\left(a_1 I - E\right)^2 \quad \text{(C2.2-16b)}$$

$$u^2(a_1) = 1 \Big/ \sum pI^2 \quad \text{(C2.2-16c)}$$

(C.2.2-16) applies for the approximated error of the reading R, and the assigned uncertainty $u\left(E_{appr}\right)$ is given by

$$u^2\left(E_{appr}\right) = a_1^2 u^2(R) + R^2 u^2(a_1) \quad \text{(C2.2-16d)}$$

c) mean gradients

In this variant the uncertainties are $u(E_j/I_j) = u(E_j)/I_j$ and $p_j = I_j^2/u^2(E_j)$.

$$a = \left(\sum pE/I\right)/\sum p \quad \text{`} \tag{C2.2-17a}$$

$$\min\chi^2 = \sum p(a - E/I)^2 \tag{C2.2-17b}$$

$$u^2(a) = 1/\sum p \tag{C2.2-17c}$$

(C.2.2-16) applies for the approximated error of the reading *R* which may be also a net indication, and the uncertainty of the approximation $u(E_{appr})$ is given by

$$u^2(E_{appr}) = a^2u^2(R) + R^2u^2(a) \tag{C2.2-17d}$$

C3 Terms without relation to the readings

While terms that are not a function of the indication do not offer any estimated value for an error to be expected for a given reading in use, they may be helpful to derive the "global uncertainty" mentioned in 7.5.2.

C3.1 Mean error

The mean of all errors is

$$E^0 = \bar{E} = \frac{1}{n}\sum_{j=1}^{n} E_j \tag{C3.1-1}$$

with the standard deviation

$$s(E) = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(\bar{E} - E_j\right)^2} = u_{appr} \tag{C3.1-2}$$

Note: the data point $I = 0$, $E = 0$ shall be included as I_1, E_1.

Where $\bar{E}$ is close to Zero, only $s^2(E)$ may be added in (7.5.2-2a). In other cases, in particular where $|\bar{E}| \geq u(W)$, (7.5.2-3a) should be used, with $u(W)$ increased by $u_{appr} = s(E)$.

C3.2 Maximum error

The "maximum error" shall be understood as the largest absolute value of all errors:

$$E_{\max} = |E_j|_{\max} \tag{C3.2-1}$$

C3.2.1 With $E^0 = E_{\max}$, (7.5.2-3a) would certainly describe a "global uncertainty" which would cover any error in the weighing range with a higher coverage probability than 95 %. The advantage is that the formula is simple and straightforward.

C3.2.2 Assuming a rectangular distribution of all errors over the – fictitious! – range $\pm E_{\max}$, E^0 could be defined as the standard deviation of the errors

$$E^0 = E_{\max}/\sqrt{3} \tag{C3.2-2}$$

to be inserted into (7.5.2-2a).

APPENDIX D: SYMBOLS AND TERMS

D1 Symbols of general application

Symbols that are used in more than one section of the main document, are listed and explained hereafter

Symbol	Definition	Unit
C	correction	
D	drift, variation of a value with time	
E	error (of an indication)	g, kg, t
I	indication of an instrument	g, kg, t
L	load on an instrument	g, kg, t
Max	maximum weighing capacity	g, kg, t
Max'	upper limit of specified ‘weighing range, $Max' < Max$	g, kg, t
Min	value of the load below which the weighing result may be subject to an excessive relative error	g, kg, t
Min'	lower limit of specified weighing range, $Min' > Min$	g, kg, t
R	indication (reading) of an instrument not related to a test load	g, kg, t
T	temperature	°C, K
Tol	specified tolerance value	
U	expanded uncertainty	g, kg, t
W	weighing result, weight in air	g, kg, t

d	scale interval, the difference in mass between two consecutive indications of the indicating device	g, kg, t
d_T	effective scale interval < d , used in calibration tests	g, kg, t
k_x	number of items x, as indicated in each case	
k	coverage factor	
M	Mass of an object	g, kg, t
m_c	conventional value of mass, preferably of a standard weight	g, kg, t
m_N	nominal conventional value of mass of a standard weight	g, kg, t
m_{ref}	reference weight ("true value") of a test load	g, kg, t
mpe	maximum permissible error (of an indication, a standard weight etc.) in a given context	g, kg
n	number of items, as indicated in each case	
s	standard deviation	
t	time	h, min
u	standard uncertainty	
$\hat{w}$	standard uncertainty related to a base quantity	

υ	number of degree of freedom	
ρ	density	kg/m³
ρ_0	reference density of air, ρ_0 = 1,2 kg/m³	kg/m³
ρ_a	air density	kg/m³
ρ_c	reference density of a standard weight, ρ_c = 8 000 kg/m³	kg/m³

-Suffix	related to
B	air buoyancy
D	drift
N	nominal value
T	test
adj	adjustment
$appr$	approximation
cal	calibration
$conv$	convection
dig	digitalisation
ecc	eccentric loading
gl	global, overall
i	numbering
$intr$	weighing instrument
j	numbering
$\max$	maximum value from a given population
$\min$	minimum value from a given population
$proc$	weighing procedure
ref	reference
rep	repeatability
s	standard (mass); actual at time of adjustment
sub	substitution load
$tare$	tare balancing operation
$temp$	temperature
$time$	time
0	zero, no-load

D2 Locations of important terms and expressions

D2.1 Calibration tests and measurement results

Quantity	Components of standard uncertainty	Sections, subsections
Indication I_j **for discrete test load** m_j		4.4.1; 6.2.1
Indication I $I = I_L + \delta I_{digL} + \delta I_{rep} + \delta I_{ecc} - I_0 - \delta I_{dig0}$ $u^2(I) = u^2(\delta I_{digL}) + u^2(\delta I_{rep}) + u^2(\delta I_{ecc})$ $+ u^2(\delta I_{dig0})$	$u(I)$ consisting of $d_0/\sqrt{12} + d_L/\sqrt{12}$ for rounding, s or s_{pool} for repeatability, $\hat{w}(I_{ecc})I$ for eccentricity of test load	**4.4; 6; 7.1** 7.1.1; 7.1.1.57.1.1.1+2 7.1.1.3 7.1.1.4
Repeatability Mean of n indications: $\bar{I}_j = \frac{1}{n}\sum_{i=1}^{n} I_{ji}$	Standard deviation: $s(I_j) = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(I_{ji} - \bar{I}_j)^2}$	**4.4; 6.1**
Eccentricity $\Delta I_{ecci} = I_i - I_1$	$\hat{w}(I_{ecc}) = \lvert \Delta I_{ecc,i} \rvert_{max} / (2L_{ecc}\sqrt{3}$	**6.3**; 7.1.1.4
Reference mass m_{ref} $m_{ref} = m_N + \delta m_c + \delta m_B + \delta m_D +$ δm_{conv} $u^2(m_{ref}) = u^2(\delta m_c) + u^2(\delta m_B) + u^2(\delta m_D)$ $+ u^2(\delta m_{conv})$ For test loads L_{Tn} comprising substitution loads: $u^2(L_{Tn}) = n^2 u^2(m_{c1}) + 2\sum u^2(I_{j-1})\Big\|_{j=1}^{n}$	$u(m_{ref})$ consisting of $u(\delta m_c)$ or $\hat{w}(m_c)$ for calibration, $\hat{w}(m_B)$ for buoyancy, $u(m_D)$ for drift, $u(m_{conv})$ for convection, $u(m_{c1}) = u(m_{ref})$ as above $u(I_{j-1}) = u(I(L_{Tj-1}))$	**4.3; 7.1** 7.1.2, 7.1.2.5 + 6 7.1.2.1 7.1.2.2, App. A + E 7.1.2.3 7.1.2.4 App. F 7.1.1.5
Error E $E = I - m_{ref}$ $u^2(E) = u^2(I) + u^2(m_{ref})$	Without convection effects: $u^2(E) = \alpha^2 + \beta^2 I^2$	6.2.1 7.1; 7.1.3
Characteristic $E_{appr} = f(I)$, based on data sets I_j, E_j, $u(E_j)$ $u(E_{appr}) = g(I)$	$u(E_{appr})$ resulting from the approximation calculus	**6.2; 7.2; App. C**
Expanded uncertainty: $U(E) = ku(E)$ with k = 2 (normal distribution) or k ≠ 2		**7.3; App. B**

D2.2 Weighing results obtained by the user of the instrument

Quantity	Components of standard uncertainty	Sections, subsections
Reading by the user: $R = R_L + \delta R_{digL} + \delta R_{rep} - R_0$ $- \delta R_{dig0} (+ \delta R_{ecc})$ $u^2(R) = u^2(\delta R_{digL}) + s^2 + u^2(\delta R_{dig0})$	as $u(I)$ above, based on d, not d_T	**7.4** 7.4.1
Error to reading: $E(R) = E(I_j)$, and $u(E_{cal})$ from calibration certificate, or by interpolation between known values, or $E_{appr} = f(I)$, approximation formula with $u[E_{appr}]$ E values rounding to d	$u(E_{cal}) = U(E_{cal})/k_{cal}$ $u[E_{appr}(R)] = f(R) = g(I)$ as above $u[E_{appr}(R)] = U[E(R)]/k_{cal}$	**7.4** 7.4.2
Weighing results W^* **based on calibration data:** $W^* = R - E$ $u^2(W^*) = u^2(R) + u^2(E)$ W **en everyday use**: $W = W^* + \delta R_{instr} + \delta R_{proc}$ $u^2(W) = u^2(W^*) + u^2(\delta R_{instr})$ $+ u^2(\delta R_{proc})$ δR_{instr} and δR_{proc} for effects of environment and handling of the instrument being different from the situation at calibration	$u(W^*)$ consisting of $u(R)$ form above $u(E(R))$ form above $u(\delta R_{instr})$ comprising $\hat{w}(R_{temp})$ for temperature $\hat{w}(R_{bouy})$ for variation of air density $\hat{w}(R_{adj})$ for long term drift $u(\delta R_{proc})$ comprising $\hat{w}(R_{Tare})$ $\hat{w}(R_{time})$ $\hat{w}(R_{ecc})$ s_{dyn}	**7.4** (7.4-1a) (7.4-2a) 7.4.1 7.4.2 (7.4.-1b); 7.4.5 (7.4-2b) 7.4.3.1 7.4.3.2 7.4.3.3 7.4.4 7.4.4.1 7.4.4.2 7.4.4.3 7.4.4.4
Expanded uncertainty: $U(W^*)$ with k = 2 (normal distribution) or $k \neq 2$ $U(W) = ku(W)$ with k = 2		7.5, App. B
Weighing result with correction: $W = R - E \pm U(W)$	$U(W)$ from above	7.5.1
Weighing result without correction: $W = R \pm U_{gl}(W)$ with $U_{gl}(W) = f\{U(W) + E(R)\}$	$U(W)$ from above, enlarged by term representing $E(R)$	7.5.2
Weighing result within specified limits: $W = R \pm Tol(R)$ with Tol specified by client, under conditions that $\lvert E(R) \rvert + U(W(R)) \leq Tol(R)$		7.5.3
Conversion of W to mass m, or to conventional value of mass m_c	To be calculated based on W by user of the instrument	**9.1** **9.2**

APPENDIX E: INFORMATION ON AIR BUOYANCY

This Appendix gives additional information to the air buoyancy correction treated in 7.1.2.2. It focusses on the standard uncertainty for the correction, as 7.1.2.2 advises to apply a correction value of $\delta m_B = 0$ with an appropriate standard deviation.

E1 Density of standard weights

Where the density ρ of a standard weight, and its standard uncertainty $u(\rho)$ are not known, the following values may be used for weights of R111 classes E2 to M2 (taken from [4], Table B7).

Alloy/material	Assumed density ρ in kg/m³	Standard uncertainty $u(\rho)$ in kg/m³
Nickel silver	8 600	85
Brass	8 400	85
stainless steel	7 950	70
carbon steel	7 700	100
iron	7 800	100
cast iron (white)	7 700	200
cast iron (grey)	7 100	300
aluminium	2 700	65

For weights with an adjustment cavity filled with a considerable amount of material of different density, [4] gives a formula to calculate the overall density of the weight.

E2 Examples for air buoyancy in general

Table E2.1 gives relative standard uncertainties for air buoyancy corrections assumed to be zero, for

- standard weights made of the alloys/materials mentioned in E1
- selected standard uncertainties of the air density – cf. the table in A3.1
- the cases A, B1, and B2 related to the adjustment of the calibrated instrument.

The formulae are (7.1.2-5) for case A, (7.1.2-7) for case B1, and (7.1.2-9) for case B2.
For case B1, $u(\delta\rho_{as}) = 0{,}5u(\rho_a)$ has been assumed.

It is obvious that for case A the relative uncertainty $\hat{w}(m_B)$ is always below 0,4 mg/kg for the materials normally used for standard weights of higher accuracy (stainless steel nowadays, previously brass), and need therefore be considered only for calibrations with extremely small uncertainty.

For case B1 calibrations, the relative uncertainty $\hat{w}(m_B)$ is below 5 mg/kg for all materials but aluminium, and for case B2 calibrations below 10 mg.

Table E2.1 Relative standard uncertainty of air buoyancy correction

$\hat{w}(m_B)$ in mg/kg for case A			ρ_a = 1,2 kg/m³ with $u(\rho_a)$ below			
Material	ρ	$u(\rho)$	0,016	0,025	0,04	0,064
nickel silver	8 600	85	0,14	0,22	0,35	0,56
brass	8 400	85	0,10	0,15	0,24	0,39
stainless steel	7 950	70	0,02	0,03	0,05	0,09
cast iron (white)	7 700	200	0,09	0,15	0,24	0,38
cast iron (grey)	7 100	300	0,27	0,42	0,68	1,08
aluminium	2 700	65	3,93	6,14	9,82	15,71

$\hat{w}(m_B)$ in mg/kg for case B1			ρ_a = 1,2 kg/m³ with $u(\rho_a)$ below			
Material	ρ	$u(\rho)$	0,016	0,025	0,04	0,064
nickel silver	8 600	85	1,01	1,58	2,52	4,04
brass	8 400	85	1,01	1,57	2,51	4,02
stainless steel	7 950	70	1,00	1,56	2,50	4,00
cast iron (white)	7 700	200	1,00	1,57	2,51	4,01
cast iron (grey)	7 100	300	1,03	1,61	2,58	4,13
aluminium	2 700	65	4,05	6,33	10,13	16,21

$\hat{w}(m_B)$ in mg/kg for case B2			ρ_a = 1,2 kg/m³ with $u(\rho_a)$ below			
Material	ρ	$u(\rho)$	0,016	0,025	0,04	0,064
nickel silver	8 600	85	1,86	2,91	4,65	7,44
brass	8 400	85	1,90	2,98	4,76	7,62
stainless steel	7 950	70	2,01	3,14	5,03	8,05
cast iron (white)	7 700	200	2,08	3,25	5,20	8,31
cast iron (grey)	7 100	300	2,26	3,52	5,64	9,02
aluminium	2 700	65	5,93	9,26	14,82	23,71

E3 Air buoyancy for weights conforming to R111

As quoted in a footnote to 7.1.2.2, R 111 requires the density of a standard weight to be within certain limits that are related to the maximum permissible error mpe and a specified variation of the air density. The mpe are proportional to the nominal value for weights of ≥ 100 g. This allows an estimate of the relative uncertainty $\hat{w}(m_B)$. The corresponding formulae (7.1.2-5a) for case A and (7.1.2-9a) for cases B1 and B2, have been evaluated in Table E2.2, in relation to the accuracy classes E_2 to M_1.

For weights of $m_N \leq$ 50 g the mpe are tabled in R111, the relative value mpe/m_N increasing with decreasing mass. For these weights, Table E2.2 contains the absolute standard uncertainties $u(m_B) = \hat{w}(m_B)m_N$.

A comparison of the relative uncertainties shows that the values from Table E2.2 are always greater than the corresponding values from Table E2.1. This is due to the fact that the assumed uncertainties $u(\rho)$ and $u(\rho_a)$ are greater in Table E2.2.

The values in Table E2.2 can be used for a "worst case" estimate of the uncertainty contribution for air buoyancy in a given situation.

Table E2.2: Standard uncertainty of air buoyancy correction for standard weights conforming to R 111

Calculated according to 7.1.2.2 for cases A (7.1.2-5a) and B (7.1.2-9a)

	Class E_2			Class F_1			Class F_2			Class M_1		
m_N in g	mpe in mg	u_A in mg	u_B in mg	mpe in mg	u_A in mg	u_B in mg	mpe in mg	u_A in mg	u_B in mg	mpe in mg	u_A in mg	u_B in mg
50	0,100	0,014	0,447	0,30	0,043	0,476	1,00	0,14	0,58	3,0	0,43	0,87
20	0,080	0,012	0,185	0,25	0,036	0,209	0,80	0,12	0,29	2,5	0,36	0,53
10	0,060	0,009	0,095	0,20	0,029	0,115	0,60	0,09	0,17	2,0	0,29	0,38
5	0,050	0,007	0,051	0,16	0,023	0,066	0,50	0,07	0,12	1,6	0,23	0,27
2	0,040	0,006	0,023	0,12	0,017	0,035	0,40	0,06	0,08	1,2	0,17	0,19
1	0,030	0,004	0,013	0,10	0,014	0,023	0,30	0,04	0,05	1,0	0,14	0,15
0,5	0,025	0,004	0,008	0,08	0,012	0,016	0,25	0,04	0,04	0,8	0,12	0,12
0,2	0,020	0,003	0,005	0,06	0,009	0,010	0,20	0,03	0,03	0,6	0,09	0,09
0,1	0,016	0,002	0,003	0,05	0,007	0,008	0,16	0,02	0,02	0,5	0,07	0,07

Relative *mpe* relative standard uncertainties $\hat{w}(m_B)$ in mg/kg for weights of 100 g and greater

	Class E_2			Class F_1			Class F_2			Class M_1		
	mpe/ mg	$\hat{w}_A$	$\hat{w}_B$	*mpe/* mg	$\hat{w}_A$	$\hat{w}_B$	*mpe/* mg	$\hat{w}_A$	$\hat{w}_B$	*mpe/* mg	$\hat{w}_A$	$\hat{w}_B$
$\geq$ 100	1,60	0,23	8,89	5,00	0,72	9,38	16,0	2,31	11,0	50,0	7,22	15,88

APPENDIX F: EFFECTS OF CONVECTION

In 4.2.3 the generation of an apparent change of mass Δm_{conv} by a difference in temperature ΔT between a standard weight and the surrounding air has been explained in principle. More detailed information is presented hereafter, to allow an assessment of situations in which the effect of convection should be considered in view of the uncertainty of calibration

All calculations of values in the following tables are based on [6]. The relevant formulae, and parameters to be included, are not reproduced here. Only the main formulae, and essential conditions are referenced.

The problem treated here is quite complex, both in the underlying physics and in the evaluation of experimental results. The precision of the values presented hereafter should not be overestimated.

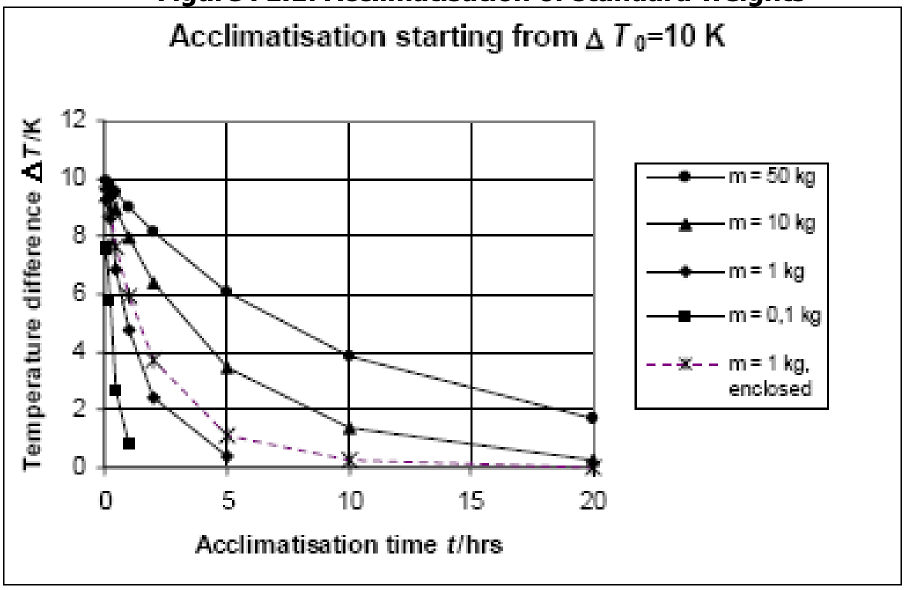
F1 Relation between temperature and time

An initial temperature difference ΔT_0 is reduced with time Δt by heat exchange between the weight and the surrounding air. The rate of heat exchange is fairly independent of the sign of ΔT_0, therefore warming up or cooling down of a weight occurs in similar time intervals.

Figure F.1.1 gives some examples of the effect of acclimatisation. Starting from an initial temperature difference of 10 K, the actual ΔT after different acclimatisation times is shown for 4 different weights. The weights are supposed to rest on three fairly thin PVC columns in "free air". In comparison, ΔT is also shown for a 1 kg weight resting on the same columns but enclosed in a bell jar which reduces the air flow of convection, so it takes about 1,5 times to 2 times as much time to achieve the same reduction of ΔT , as for the 1 kg piece without the jar.

References in [6]: formula (21), and parameters for cases 3b and 3c in Table 4.

Figure F1.1: Acclimatisation of standard weights



Tables F1.2 and F1.3 give acclimatisation times Δt for standard weights that may have to be waited if the temperature difference is to be reduced from a value ΔT_1 to a lower value ΔT_2. The conditions of heat exchange are the same as in Figure F1.1: Table F1.2 as for "m = 0,1 kg" to "m = 50 kg"; Table F1.3 as for "m = 1 kg enclosed".

Under actual conditions the waiting times may be shorter where a weight stands directly on a plane surface of a heat conducting support; they may be longer where a weight is partially enclosed in a weight case.
References in [6]: formula (26), and parameters for cases 3b, 3c in Table 4.

Table F1.2 Time intervals for reduction in steps of temperature differences
Weights standing on 3 thin PVC columns in free air

Acclimatisation time in *min* for ΔT to be reached from the next higher ΔT, Case 3b								
	ΔT / K							
m/kg	20	15	10	7	5	3	2	1
50		149,9	225,3	212,4	231,1	347,9	298,0	555,8
20		96,2	144,0	135,2	135,0	219,2	186,6	345,5
10		68,3	101,9	95,3	94,8	153,3	129,9	239,1
5		48,1	71,6	66,7	66,1	106,5	89,7	164,2
2		30,0	44,4	41,2	40,6	65,0	54,4	98,8
1		20,8	30,7	28,3	27,8	44,3	37,0	66,7
0,5		14,3	21,0	19,3	18,9	30,0	24,9	44,7
0,2		8,6	12,6	11,6	11,3	17,8	14,6	26,1
0,1		5,8	8,5	7,8	7,5	11,8	9,7	17,2
0,05		3,9	5,7	5,2	5,0	7,8	6,4	11,3
0,02		2,3	3,3	3,0	2,9	4,5	3,7	6,4
0,01		1,5	2,2	2,0	1,9	2,9	2,4	4,2

Examples for a 1 kg weight:
- to reduce ΔT from 20 K to 15 K will take 20,8 min;
- to reduce ΔT from 15 K to 10 K will take 30,7 min;
- to reduce ΔT from 10 K to 5 K will take 28,3 min + 27,8 min = 56,1 min

Table F1.3 Time intervals for reduction in steps of temperature differences
Weights standing on 3 thin PVC columns, enclosed in a bell jar

Acclimatisation time in *min* for ΔT to be reached from the next higher ΔT, Case 3c								
	ΔT / K							
m/kg	20	15	10	7	5	3	2	1
50		154,2	235,9	226,9	232,1	388,7	342,7	664,1
20		103,8	158,6	152,4	155,6	260,2	228,9	442,2
10		76,8	117,2	112,4	114,7	191,5	168,1	324,0
5		56,7	86,4	82,8	84,3	140,5	123,1	236,5
2		37,8	57,5	54,9	55,8	92,8	81,0	155,0
1		27,7	42,1	40,1	40,7	67,5	58,8	112,0
0,5		20,2	30,7	29,2	29,6	49,9	42,4	80,5
0,2		13,3	20,1	19,1	19,2	31,7	27,3	51,6
0,1		9,6	14,5	13,7	13,8	22,6	19,5	36,6
0,05		6,9	10,4	9,8	9,9	16,1	13,8	25,7
0,02		4,4	6,7	6,3	6,2	10,2	8,6	16,0
0,01		3,2	4,7	4,4	4,4	7,1	6,0	11,1

F2 Change of the apparent mass

The air flow generated by a temperature difference ΔT is directed upwards where the weight is warmer -$\Delta T > 0$-, than the surrounding air, and downwards where it is cooler -$\Delta T < 0$. The air flow causes friction forces on the vertical surface of a weight, and pushing or pulling forces on its horizontal surfaces, resulting in a change Δm_{conv} of the apparent mass. The load receptor of the instrument is also contributing to the change, in a manner not yet fully investigated.

There is evidence from experiments that the absolute values of the change are generally smaller for $\Delta T < 0$ than for $\Delta T > 0$. It is therefore reasonable to calculate the mass changes for the absolute values of ΔT, using the parameters for $\Delta T > 0$.

Table F2.1 gives values for Δm_{conv} for standard weights, for the temperature differences ΔT appearing in Tables F1.2 and F1.3. They are based on experiments performed on a mass comparator with turning table for automatic exchange of weights inside a glass housing. The conditions prevailing at calibration of "normal" weighing instruments being different, the values in the table should be considered as estimates of the effects that may be expected at an actual calibration.
References in [6]: formula (34), and parameters for case 3d in Table 4

Table F2.1 Change in apparent mass Δm_{conv}

Change Δm_{conv} in mg of standard weights, for selected temperature differences ΔT								
	ΔT in K							
m in kg	20	15	10	7	5	3	2	1
50	113,23	87,06	60,23	43,65	32,27	20,47	14,30	7,79
20	49,23	38,00	26,43	19,25	14,30	9,14	6,42	3,53
10	26,43	20,47	14,30	10,45	7,79	5,01	3,53	1,96
5	14,30	11,10	7,79	5,72	4,28	2,76	1,96	1,09
2	6,42	5,01	3,53	2,61	1,96	1,27	0,91	0,51
1	3,53	2,76	1,96	1,45	1,09	0,72	0,51	0,29
0,5	1,96	1,54	1,09	0,81	0,61	0,40	0,29	0,17
0,2	0,91	0,72	0,51	0,38	0,29	0,19	0,14	0,08
0,1	0,51	0,40	0,29	0,22	0,17	0,11	0,08	0,05
0,05	0,29	0,23	0,17	0,12	0,09	0,06	0,05	0,03
0,02	0,14	0,11	0,08	0,06	0,05	0,03	0,02	0,01
0,01	0,08	0,06	0,05	0,03	0,03	0,02	0,01	0,01

The values in this table may be compared with the uncertainty of calibration, or with a given tolerance of the standard weights that are used for a calibration, in order to assess whether an actual ΔT value may produce a significant change of apparent mass.

As an example, Table F2.2 gives the temperature differences which are likely to produce, for weights conforming to R 111, values of Δm_{conv} not exceeding certain limits. The comparison is based on Table F2.1.
The limits considered are the maximum permissible errors, or $1/3$ thereof.

It appears that with these limits, the effect of convection is relevant only for weights of classes E_2 and F_1 of R111.

Table F2.2 Temperature limits for specified Δm_{conv} values

ΔT_A = temperature difference for $\Delta m_{conv} \leq mpe$

ΔT_B = temperature difference for $\Delta m_{conv} \leq mpe/3$

Differences ΔT_A for $\Delta m_{conv} < mpe$ and ΔT_B for $\Delta m_{conv} < mpe/3$						
	Class E_2			Class F_1		
m_N in kg	mpe in mg	ΔT_A in K	ΔT_B in K	mpe in mg	ΔT_A in K	ΔT_B in K
50	75	12	4	250	>20	12
20	30	7	3	100	>20	7
10	15	10	3	50	>20	10
5	7,5	10	3	25	>20	10
2	3	9	1	10	>20	9
1	1,5	7	1	5	>20	7
0,5	0,75	6	1	2,5	>20	6
0,2	0,30	5	1	1,0	>20	5
0,1	0,15	4	1	0,50	>20	4
0,05	0,10	6	1	0,30	>20	6
0,02	0,08	10	2	0,25	>20	10
0,01	0,06	15	3	0,20	>20	15

APPENDIX G: EXAMPLES

The examples presented in this Appendix demonstrate in different ways how the rules contained in this guideline may be applied correctly. They are not intended to indicate any preference for certain procedures as against others for which no example is presented.
Where a calibration laboratory wishes to proceed in full conformity to one of them, it may make reference to it in its quality manual and in any certificate issued.

Note 1: The certificate should contain all the information presented in Gn.1, as far as known, and, as applicable, at least what is printed in bold figures in Gn.2 and Gn.3, with Gn = G1, G2…
Note 2: For references to the relevant sections of the guideline see Appendix D2.

G1 Instrument of 200 g capacity, scale interval 0,1 mg

G1.1 Conditions specific for the calibration

Instrument:	**electronic weighing instrument, description and identification**
Max/d	**200 g / 0,1 mg**
Temperature coefficient	$TC \leq 1{,}5x10^{-6}/\mathrm{K}$ (manufacturer's manual)
Built-in adjustment device	acts automatically upon: switch-on, and when ΔT ≥ 3 K
adjustment by calibrator	**performed before the calibration**
Temperature during calibration	**20,2 °C to 20,6 °C**
room conditions	temperature stabilized to 21 °C ± 1 °C; h ≈ 300 m
load receptor	diameter 80 mm
Test loads	**standard weights, class E_2**

G1.2 Tests and results

Repeatability (assumed to be const. over the weighing range)	**test load 100 g, applied 6 times**, indication at no load reset to zero where necessary; indications recorded: 100,000 2 g; 99,999 9 g; 100,000 1 g; 100,000 0 g; 100,000 2 g; 100,000 2 g	
Errors of indication	test loads each applied once; discontinuous loading only upwards, indication at no load reset to zero where necessary; all loads in centre of load receptor. Indications recorded:	
	load/g	indication/g
	30	30,000 1
	60	60,000 3
	100	100,000 4
	150	150,000 6
	200	200,000 9

Eccentricity test	**test load 100 g**; indication at no load reset to zero where necessary; positions/readings in g:
	1/100,000 5; 2/100,000 3; 3/100,000 4; 4/100,000 6; 5/100,000 4 $\left\|\Delta I_{ecc}\right\|_{\max} =$ **0,2 mg**

G1.3 Errors and related uncertainties

The calculations follow 7.1 to 7.3

Quantity or Influence	Load, indication in g Standard uncertainty in mg					distribution / degrees of freedom
Indication $I \approx m_N$ / g	**30**	**60**	**100**	**150**	**200**	
Error *Ecal* / mg	**0,1**	**0,3**	**0,4**	**0,6**	**0, 9**	
Repeatability *s*	**0,13 mg**					norm/5
Digitalisat'n $d_0/\sqrt{12}$	0,03 mg					rect/100[10]
Digitalisat'n $d_I/\sqrt{12}$	0,03 mg					rect/100
Eccentricity $\hat{w}_{ecc}(I)$	not relevant in this case					rect/100
u(I)	0,14 mg					
Test loads m_N/g [11]	10 + 20	10 + 50	100	50 + 100	200	
$u(\delta m_c) = mpe/\sqrt{3}$ / mg	0,08	0,09	0,09	0,15	0,17	rect/100
$u(\delta m_D) = mpe/(3\sqrt{3})$ / mg	0,03	0,03	0,03	0,05	0,06	rect/100
$\hat{w}(m_B)m_N = mpe/(4\sqrt{3})$ / mg	0,02	0,02	0,02	0,04	0,04	rect/100
.. $u(\delta m_{conv})$ / mg	not relevant in this case					
Unc. of error $u(E)$ / mg	0,165	0,170	0,170	0,215	0,232	
υ_{eff}	12	14	14	34	44	
k (95,45 %)	2,23	2,20	2,20	2,08	2,06	
***U(E) = ku(E)* / mg**	**0,37**	**0,37**	**0,37**	**0,45**	**0,48**	
additional, optional						
Approximation by straight line through zero / mg	$E_{appr}(R) = 4{,}27\times10^{-6}R$					
Uncertainty to approximated errors, $u(E_{appr})$ / mg	$u(E_{appr}) = \sqrt{(3{,}25\times10^{-13}\,\mathrm{mg}^2 + 5{,}8\times10^{-13}R^2)}$ [12]					
Expanded uncertainty $U(E_{appr})$ / mg	$U(E_{appr}) = 2\sqrt{(3{,}25\times10^{-13}\,\mathrm{mg}^2 + 5{,}8\times10^{-13}R^2)} = 1{,}5\times10^{-6}R$					

It would be acceptable to state in the certificate only the largest value of expanded uncertainty for all the reported errors: $U(E)$ **= 0,48 mg**, based on ***k* = 2,06 for υ_{eff} = 44**, accompanied by the statement that the coverage probability is at least 95 %.

The certificate may give the advice to the user that the standard uncertainty to the error of any reading *R*, obtained after the calibration, is increased by the uncertainty of the reading *u(R)* = 0,14 mg.

[10] There were assumed 100 degrees of fredom for all type B uncertianties, $\Delta u(x_i)/u(x_i) \approx 0{,}0707$ [1]

[11] Class E_2, calibrated three months ago, average drift monitored over two recalibrations $|D_{mc}| \leq mpe/3$; over 12 months; used at nominal value; well accomodated to room temperature,, ΔT < 1 K

[12] The first term is negligible!

G1.4 Uncertainty of indications in use

As stated in 7.4, the following information may be developed by the calibration laboratory or by the user of the instrument. In any case, it may not be presented nor considered as part of the calibration certificate.

G1.4.1 The normal conditions of use of the instrument, as assumed, or as specified by the user may include
Variation of temperature ± 1 K
Loads not always centered carefully
Tare balancing function operated
Loading times: normal, as at calibration

G1.4.2 Calculation table as per 7.4 and 7.5

Quantity or Influence	Indication in g Error, uncertainty: relative or in mg					Distribution / degrees of freedom
Indication $I \approx m_N / g$	30	60	100	150	200	
Error E_{cal} / mg	0,1	0,3	0,4	0,6	0,9	
Uncertainty $u(E)$	0,23 mg					norm/44
Alternative: to quote results of the approximation						
Error E_{appr} / mg	$4{,}27 \times 10^{-6} R$					
$u(E_{appr})$ / mg	$0{,}76 \times 10^{-6} R$					
Repeatability s_R	0,13 mg					norm/5
Digitalisat'n $d_0 / \sqrt{12}$	0,03 mg					rect/100
Digitalisat'n $d_R / \sqrt{12}$	0,03 mg					rect/100
Adjustm't drift $\hat{w}(R_{adj})$	not relevant in this case, as instrument is regularly adjusted					
Temperature $\hat{w}(R_{temp})$	$0{,}87 \times 10^{-6}$					rect/100
Weighing. procedure: $\hat{w}(R_{ecc})$	$1{,}15 \times 10^{-6}$					rect/100
$\hat{w}(R_{tare})$	$1{,}23 \times 10^{-6}$					rect/100
$\hat{w}(R_{time})$	not relevant in this case					
Unc. of weighing result $u(W)$	$u(W) = \sqrt{\left(0{,}0178\ \text{mg}^2 + 4{,}0 \times 10^{-12} R^2\right)}$					
υ_{eff}	> 30					
$k(\approx 95\%)$	2					
Uncertainty of weighing result with correction by - E_{appr}						
$U(W) = ku(W)$	$U(W) = 2\sqrt{0{,}0178\ \text{mg}^2 + 4{,}0 \times 10^{-12} R^2}$					
simplified to first order	$U(W) \approx U(W=0) + \left\{ \begin{bmatrix} U(W = Max) - \\ U(W=0) \end{bmatrix} / \right.$ $U(W) \approx 0{,}27\text{mg} + 2{,}88 \times 10^{-6} R$					
Global uncertainty of weighing result without correction to the reading						
$U_{gl}(W) = U(W) + \lvert E_{appr}(R) \rvert$	$U_{gl}(W) = 0{,}27\ \text{mg} + 7{,}15 \times 10^{-6} R$					

G1.4.3 An attachment to the certificate could contain this statement:

"Under normal conditions of use, including
room temperature varying within ± 1 K,
loads applied without special care to apply centre of gravity in centre of load receptor,
obtaining readings *R* with or without tare balancing (Net or Gross values),
enabling automatic adjustment of the instrument,
not applying any correction to the readings *R*,

the weighing result *W* is

$$W = R \pm \left(0{,}27mg + 7{,}1 \times 10^{-6} R\right)$$

at a level of confidence of better than 95%."

An alternative would read:

(Conditions as before)..".., the weighing result W is
within a tolerance of 1 % for $R \geq 30$ mg,
within a tolerance of 0,1 % for $R \geq 280$ mg,
at a level of confidence of better than 95%."

G2 Instrument of 60 kg, multi-interval

G2.1 Conditions specific for the calibration

Instrument:	**electronic weighing instrument, description and identification**, with OIML R76 (or EN 45501) type approval, but not verified
Max/d	**Multi-interval instrument, 3 partial weighing ranges:** Max_i/kg **= 12/30/60;** d_i/g **= 2/5/10**
Load receptor	platform 60 cm × 40 cm
Installation	In packaging workroom; 17 °C ≤ *T* ≤ 27 °C reported by client
Temperature coefficient	$TC \leq 2 \times 10^{-6}$ /K (manufacturer's manual)
Built-in adjustment device	not provided; $\lvert E(Max)\rvert \leq 10g$ (manufacturer's manual)
Last calibration	Performed 1 year ago; $E(Max)$ was 7 g
Temperature during calibration	**22,3 °C to 23,1 °C**
Barometric pressure during calibration:	1 002 hPa ± 5 hPa
Test loads	**standard weights**, stainless steel, certified to class M_1 tolerances of 50 mg/kg (OIML R111)

G2.2 Tests and results

<table>
<tr><td>Repeatability
(assumed to be constant over weighing range 1)</td><td colspan="2">Test load 10 kg, applied 5 times, indication at no load reset to zero where necessary. Readings recorded:
9,998 kg; 10,000 kg; 9,998 kg; 10,000 kg; 10,000 kg</td></tr>
<tr><td>Repeatability
(assumed to be constant over weighing ranges 2 and 3)</td><td colspan="2">Test load 30 kg, applied 5 times, indication at no load reset to zero where necessary. Readings recorded:
29,995 kg; 30,000 kg; 29,995 kg; 29,995 kg; 30,000 kg</td></tr>
<tr><td rowspan="10">Errors of indication</td><td colspan="2">test loads each applied once; discontinuous loading only upwards, indication at no load reset to zero where necessary; all loads in centre of load receptor.
Indications recorded:</td></tr>
<tr><td>Load / kg</td><td>Indication / kg</td></tr>
<tr><td colspan="2">without tare load</td></tr>
<tr><td>10</td><td>10,000</td></tr>
<tr><td>25</td><td>24,995</td></tr>
<tr><td>40</td><td>39,990</td></tr>
<tr><td>60</td><td>59,990</td></tr>
<tr><td colspan="2">25 kg put on load receptor, indication set to Net zero by tare operation</td></tr>
<tr><td>10</td><td>9,998</td></tr>
<tr><td>20</td><td>19,995</td></tr>
<tr><td>Eccentricity test</td><td colspan="2">test load 20 kg; indication at no load reset to Zero where necessary; positions/readings:</td></tr>
<tr><td></td><td colspan="2">1: 19,995 kg; 2: 19,995 kg; 3: 19,995 kg
4: 19,990 kg; 5: 19,990 kg;
$\left|\Delta I_{ecc}\right|_{\max}$ = 5 g</td></tr>
</table>

G2.3 Errors and related uncertainties

The calculations follow 7.1 to 7.3

Quantity or Influence	Load, indication, error in kg Standard uncertainty in g, or as relative value						Distribution / degrees of freedom
Indication $I \approx m_N$ / kg	**10**	**25**	**40**	**60**			
Error *Ecal* / kg	**0**	**-0,005**	**-0,010**	**-0,010**			
Indication I_{Net} / kg	**after tare balancing a preload of 25 kg**				**10**	**20**	
Error $E_{Cal,Net}$ / kg					**-0,002**	**-0,005**	
Repeatability *s* / g	**1,10**	**2,74**			**1,10**	**2,74**	norm/4
Digitalisat'n $d_0/\sqrt{12}$	0,58						rect/100
Digitalisat'n $d_I/\sqrt{12}$	0,58	1,44	2,89	2,89	0,58	1,44	rect/100
Eccentricity $\hat{w}_{ecc}(I)$	not relevant in this case						rect/100
u(I) / g	1,37	3,15	4,02	4,02	1,37	3,15	
Test loads [13] / kg	10	20+5	2*20	3*20	(25+) 10	(25+) 20	
$u(\delta m_c) = mpe/\sqrt{3}$	0,29	0,72	1,15	1,73	0,29	0,58	rect/100
$u(\delta m_D) = mpe/(2\sqrt{3})$	0,14	0,36	0,58	0,87	0,14	0,29	rect/100
$u(\delta m_B) = \hat{w}(m_B) m_N$ =(2,6 mg/kg) m_N [14]	negligible						
$u(\delta m_{conv})$ / g	negligible						
Unc. of error $u(E)$	1,41	3,25	4,22	4,46	1,41	3,22	
υ_{eff}	10	7	21	26	10	7	
k (95,45 %)	2,28	2,43	2,13	2,10	2,28	2,43	
***U(E) = ku(E)* / g**	**3,2**	**7,9**	**9,0**	**9,4**	**3,2**	**7,8**	
Approximation, performed with the 4 gross indications							
Approximation by straight line through zero / kg	$E_{appr}(R) = -1{,}69 \times 10^{-4} R$						
Uncertainty to approximated errors, $u(E_{appr})$, for partial weighing ranges (*PWR*)	*PWR 1*	$u(E_{appr}) = \sqrt{(5{,}4 \times 10^{-8}\,\mathrm{g}^2 + 2{,}63 \times 10^{-9} R^2)}$ [15]					
	PWR 2	$u(E_{appr}) = \sqrt{(2{,}8 \times 10^{-7}\,\mathrm{g}^2 + 2{,}63 \times 10^{-9} R^2)}$					
	PWR 3	$u(E_{appr}) = \sqrt{(4{,}7 \times 10^{-7}\,\mathrm{g}^2 + 2{,}63 \times 10^{-9} R^2)}$					
$u(E_{appr})$, for *PWR* 1 to 3	$u(E_{appr}) = 5{,}13 \times 10^{-5} R$						
Expanded uncertainty, with *k*=2	$U(E_{appr}) = 2u(E_{appr}) = 10{,}3 \times 10^{-5} R$						

13 Class M_1, calibrated 8 months ago, average drift monitored over 2 recalibrations: $|D_{mc}| \leq mpe/2$; over 12 months; used at nominal value; well accomodated to room temperature, ΔT < 1 K

14 For Δp =40 hPa, ΔT =10 K, $u(\rho_a)$=0,0207 kg/m³ (from table in A3.1)

15 The first term is negligible in all 3*PWR!*

Quantity or Influence	Load, indication, error in kg Standard uncertainty in g, or as relative value	Distribution / degrees of freedom
For comparison's sake, approximations repeated with all 6 indications		
Approximation by straight line through zero	$E_{appr}(R) = -1{,}79 \times 10^{-4}\, R$	
$u(E_{appr})$, for *PWR* 1 to 3	$u(E_{appr}) = 4{,}62 \times 10^{-5}\, R$	
Expanded uncertainty, with *k*=2 / g	$U(E_{appr}) = 2u(E_{appr}) = 9{,}2 \times 10^{-5}\, R$	
At Max, the first approximation gives $E = -10{,}1$ g, $U(E_{appr}) = 6{,}2$ g; the second approximation gives $E = -10{,}7$ g, $U(E_{appr}) = 5{,}5$ g: The differences are not significant. see G2.5.1		

It would be acceptable to state in the certificate only the largest value of expanded uncertainty for all the reported errors: $\boldsymbol{U(E)}$ = **9,4 g**, based on ***k* = 2,1** for $\boldsymbol{\upsilon_{eff}}$ = **26,** accompanied by the statement that the coverage probability is at least 95 %.

The certificate may give the advice to the user that the standard uncertainty to the error of any reading R, obtained after the calibration, is increased by the standard uncertainty of the reading $u(R)$ depending on the scale interval:

from 0 to 12 kg: *d* = 2 g, *u(R)* = 1,4 g

from 12 to 30 kg: *d* = 5 g, *u(R)* = 3,2 g

from 30 to 60 kg: *d* = 10 g, *u(R)* = 4,0 g

For the test points mentioned above, the uncertainties $U(W^*)$ to the weighing results under the conditions of the calibration: $W^* = R - E$, are then

Reading *R*/kg	10	25	40	60
Uncertainty *U(W*)*/g	3,9	9,1	11,7	12,0

G2.4 Uncertainty of indications in use

As stated in 7.4, the following information may be developed by the calibration laboratory or by the user of the instrument. In any case, it may not be presented nor considered as part of the calibration certificate.

G2.4.1 The normal conditions of use of the instrument, as assumed, or as specified by the user may include

Variation of temperature ± 5 K

Loads not always centered carefully

Tare balancing function operated

Loading times: normal, as at calibration

G2.4.2 Calculation table as per 7.4 and 7.5

<table>
<tr><td>Quantity or Influence</td><td colspan="3">Load, indication, error in kg Standard uncertainty in g, or as relative value</td><td>Distribution / degrees of freedom</td></tr>
<tr><td>Errors of indication for gross and net readings</td><td colspan="3">$E_{appr}(R) = -1{,}79 \times 10^{-4}\, R$</td><td></td></tr>
<tr><td>Uncertainty to errors $u(E_{appr}(R))$</td><td colspan="3">$u(E_{appr}) = 4{,}62 \times 10^{-5}\, R$</td><td></td></tr>
<tr><td rowspan="2">Uncertainty of reading $u(R) = u(I)$</td><td>PWR 1</td><td>PWR 2</td><td>PWR 3</td><td rowspan="2"></td></tr>
<tr><td>1,37</td><td>3,15</td><td>4,02</td></tr>
<tr><td rowspan="3">Uncertainty of error $u(E(R)) = \sqrt{\{u^2(R) + u^2(E_{appr})\}}$</td><td>PWR 1</td><td colspan="2">$u(E(R)) = \sqrt{\{1{,}88\text{g}^2 + 2{,}13 \times 10^{-9}\, R^2\}}$</td><td></td></tr>
<tr><td>PWR 2</td><td colspan="2">$u(E(R)) = \sqrt{\{9{,}92\text{g}^2 + 2{,}13 \times 10^{-9}\, R^2\}}$</td><td></td></tr>
<tr><td>PWR 3</td><td colspan="2">$u(E(R)) = \sqrt{\{16{,}16\text{g}^2 + 2{,}13 \times 10^{-9}\, R^2\}}$</td><td></td></tr>
<tr><td colspan="5">Influences from instrument</td></tr>
<tr><td>Adjustment drift see G2.5.2</td><td colspan="3">$\hat{w}(R_{adj}) = 9{,}6 \times 10^{-5}$</td><td></td></tr>
<tr><td>Temperature see G2.5.3</td><td colspan="3">$\hat{w}(R_{temp}) = 5{,}8 \times 10^{-6}$</td><td></td></tr>
<tr><td colspan="5">Influences from weighing procedure</td></tr>
<tr><td>Eccentric loading</td><td colspan="3">$\hat{w}(R_{ecc}) = 1{,}44 \times 10^{-4}$</td><td></td></tr>
<tr><td>Operation of tare device</td><td colspan="3">$\hat{w}(R_{tare})$: included by calibration procedure</td><td></td></tr>
<tr><td>Loading time</td><td colspan="3">not relevant in this case</td><td></td></tr>
<tr><td rowspan="3">Uncertainty of weighing result $u(W)$</td><td>PWR 1</td><td colspan="2">$u(W) = \sqrt{\{1{,}88\text{g}^2 + 3{,}0 \times 10^{-8}\, R^2\}}$</td><td></td></tr>
<tr><td>PWR 2</td><td colspan="2">$u(W) = \sqrt{\{9{,}92\text{g}^2 + 3{,}0 \times 10^{-8}\, R^2\}}$</td><td></td></tr>
<tr><td>PWR 3</td><td colspan="2">$u(W) = \sqrt{\{16{,}20\text{g}^2 + 3{,}0 \times 10^{-8}\, R^2\}}$</td><td></td></tr>
<tr><td colspan="5">Uncertainty of weighing result with correction by E_{appr}</td></tr>
<tr><td rowspan="3">Expanded uncertainty $U(W) = ku(W)$, k=2</td><td>PWR 1</td><td colspan="2">$U(W) = 2\sqrt{\{1{,}88\text{g}^2 + 3{,}0 \times 10^{-8} R^2\}}$</td><td></td></tr>
<tr><td>PWR 2</td><td colspan="2">$U(W) = 2\sqrt{\{9{,}92\text{g}^2 + 3{,}0 \times 10^{-8} R^2\}}$</td><td></td></tr>
<tr><td>PWR 3</td><td colspan="2">$U(W) = 2\sqrt{\{16{,}20\text{g}^2 + 3{,}0 \times 10^{-8} R^2\}}$</td><td></td></tr>
<tr><td rowspan="3">simplified to first order: $U(W) \approx U(Max_{i-1}) + \left\{\frac{[U(Max_i) - U(Max_{i-1})]}{Max_i - Max_{i-1}}\right\} R$</td><td>PWR 1</td><td colspan="2">$U(W) \approx 2{,}7\text{g} + 1{,}9 \times 10^{-4}\, R$</td><td></td></tr>
<tr><td>PWR 2</td><td colspan="2">$U(W) \approx 7{,}5\text{g} + 3{,}2 \times 10^{-4}(R - 12kg)$</td><td></td></tr>
<tr><td>PWR 3</td><td colspan="2">$U(W) \approx 13{,}1\text{g} + 3{,}4x10^{-4}(R - 30kg)$</td><td></td></tr>
<tr><td colspan="5">Global uncertainty of weighing result without correction to the readings</td></tr>
<tr><td rowspan="3">$U_{gl}(W) =$
$U(W) + \left|E_{appr}(R)\right|$
also simplified to first order:</td><td>PWR 1</td><td colspan="2">$U_{gl}(W) \approx 2{,}7\text{g} + 2{,}82 \times 10^{-4} R$</td><td></td></tr>
<tr><td>PWR 2</td><td colspan="2">$U_{gl}(W) \approx 7{,}5\text{g} + 4{,}12 \times 10^{-4}(R - 12kg)$</td><td></td></tr>
<tr><td>PWR 3</td><td colspan="2">$U_{gl}(W) \approx 13{,}1\text{g} + 4{,}32 \times 10^{-4}(R - 30kg)$</td><td></td></tr>
</table>

G2.4.3 An attachment to the certificate could contain this statement:
"Under normal conditions of use, including
room temperature varying within 17 °C y 27 °C,
loads applied without special care to apply centre of gravity in centre of load receptor,
obtaining readings *R* with or without tare balancing (Net or Gross values),
not applying any correction to the readings *R*,
the weighing result *W* is
$W = R \pm U(W)$, with $U(W)$ as tabled below

Weighing range	Reading *R* from	to	Uncertainty $U(W)$ of weighing result *W* / g
PWR 1	0	12 kg	$\approx 2{,}7\,\mathrm{g} + 2{,}82 \times 10^{-4} R$
PWR 2	12 kg	30 kg	$\approx 7{,}5\,\mathrm{g} + 4{,}12 \times 10^{-4} (R - 12kg)$ $\approx 3\,\mathrm{g} + 4 \times 10^{-4} R$
PWR 3	30 kg	60 kg	$\approx 13{,}1\,\mathrm{g} + 4{,}32 \times 10^{-4} (R - 30kg)$ $\approx 4{,}4 \times 10^{-4} R$

at a level of confidence better than 95%."

An alternative would read:

(Conditions as before).."..., the weighing result *W* is
within a tolerance of 1 % for $R \geq$ 0, 28 kg,
within a tolerance of 0,5 % for $R \geq$ 0,57 kg,
within a tolerance of 0,2 % for $R \geq$ 1,56 kg,
within a tolerance of 0,1 % for $R \geq$ 3,72 kg
at a level of confidence better than 95%."

G2.5 Further information to the example

Multi-interval instruments have a varying scale interval over the weighing range – see the specifications for *Max* and *d* in G2.1 – and they display net indications after a tare balancing operation always starting with the smallest scale interval, in the same manner as they display gross indications.

It is not possible to test, with reasonable effort, such an instrument for errors of net indication with a large variety of tare loads. One may therefore consider that on an instrument with sufficient linearity of the relation $I = f(m)$, the same net load will be indicated with almost the same error irrespective of the value of the balanced tare. The approximation by a linear function without an offset value i.e. a linear function through zero - $l(m = 0) = 0$ - as per (C2.2-16) allows to assess the linearity of the relation: as long as the condition (C2.2-2), the $\min \chi^2$ criterion is satisfied by the actual test data, the approximation by the linear function is considered to be a suitable approach which means that the individual errors are indeed sufficiently close to a straight line through zero.

A test with one or two net loads applied after tare balancing a considerable preload should however, be performed to make sure that the errors for net loads are not significantly influenced by creep and hysteresis effects. As long as the errors for the same net loads, with or without a preload are about the same within the repeatability standard deviation, it may be assumed that the errors determined by the calibration actually do apply to all indicated gross or net loads.

G.2.5.1 Comparison of the approximations

This comparison shows that in this case, the error values found at test points 5 and 6 do not significantly alter the results of the approximation.

The $\min \chi^2$ values obtained by the evaluations are

2,08 to be assessed against the criterion value of **4,9** for the first approximation, and
2,33 to be assessed against the criterion value **4,9** for the second approximation.

In both cases there is no doubt that the linear approximation model is consistent with the actual test data.

G2.5.2 As stated in G2.1, the error at *Max* was +7 g at the time of the last calibration, and it is −10 g at this time. Both values are within the manufacturer's specification for the error at *Max*. With (7.4.3-2) the relative uncertainty for variation of adjustment is

$$\hat{w}\left(R_{adj}\right) = \left|\Delta E\left(Max\right)\right| / \left(Max\sqrt{3}\right) = 9{,}6 \times 10^{-5}$$

G2.5.3 As stated in G2.1, the ambient temperature about the instrument is from 17 °C to 27 °C which leads to ΔT = 10 K. The temperature coefficient of the instrument is specified by the manufacturer to be $TC \leq 2\times10^{-6}$/K: Therefore (7.4.3-1) gives

$$\hat{w}\left(R_{temp}\right) = 2x10^{-6} \times 10 / \sqrt{12} = 5{,}8 \times 10^{-6}$$

G3 Instrument of 30 t capacity, scale interval 10 kg

G3.1 Conditions specific for the calibration

Instrument:	**electronic weighing instrument, description and identification**, with OIML R76 (or EN 45501) Type approval but not verified
Max/d	**30 t / 10 kg**
load receptor	3 m wide, 10 m long, 4 points of support
Installation	Outside, in plain air, under shadow
Temperature coefficient	$TC \leq 2 \times 10^{-6}$/K (manufacturer's manual)
Built-in adjustment device	Not provided
last calibration	performed 10 months ago; error at Max was -5 kg
Scale interval for testing	**Higher resolution (service mode), d_T = 1 kg**
Duration of tests	from 9h00 to 11h00
Temperature during calibration:	**17°C to 20°C**
Barometric pressure during calibration:	**1 010 hPa ± 10 hPa**
Test loads	**Standards weights:** • **12 rollable cylindrical weights, cast iron, 500 kg each**, certified to class M_1 tolerance of mpe = 25 g (OIML R111) **Substitution loads made up of steel or cast iron:** • 6 steel containers filled with loose steel or cast iron, each weighing ≈ 3 000 kg; • trailer to support the steel containers, weight adjusted to ≈ 6 000 kg; • forklift, weight ≈ 4,5 t, capacity 6 t to move substitution loads

G3.2 Tests and results

Repeatability (assumed to be const. over the weighing range)	Fork lift with 2 steel containers, moved on alternating from either long end of load receptor, load centered by eyesight; indication at no load reset to zero where necessary. Test load ≈ 10,5 t Readings recorded: 10 411 kg; 10 414 kg; 10 418 kg; 10 412 kg; 10 418 kg. After unloading, no-load indications were between 0 and 2 kg
Errors of indication	test loads built up by substitution, with 6 000 kg standard weights and 4 substitution loads of approximately 6 t each. All test loads applied once, discontinuous loading, only upwards; indications after removal of standard weights recorded but no correction applied; all loads arranged reasonably around centre of load receptor. Indications recorded::

Load L_{Tj} / kg	Indication I_j / kg
6 000	6 001
12 014	12 014
17 996	17 999
24 014	24 019
30 001	30 010
0	4

See G3.5.1 for full record of data

Eccentricity test	Same test load of ≈ 10,5 t was used as for the repeatability test, indication at no load reset to zero where necessary; positions/readings in kg: 1/10 471 kg; 2/10 467 kg; 3/10 473 kg; 4/ 10 476 kg; 5/10 475 kg $\lvert \Delta I_{ecc} \rvert_{\mathbf{max}}$ **= 5 kg**

G3.3 Errors and related uncertainties

The calculations follow 7.1 to 7.3

Quantity or Influence	Load, indication, error in kg Standard uncertainty in kg, or as relative value					distribution / degrees of freedom
Indication $I \approx m_N$ **/ kg**	**6 000**	**12 000**	**18 000**	**24 000**	**30 000**	
Error E_{cal} **/ kg**	**1**	**0**	**3**	**5**	**9**	
Repeatability s **/ kg**	**3,3**					norm/4
Digitalisat'n $d_{T0}/\sqrt{12}$	0,3					rect/100
Digitalisat'n $d_{TI}/\sqrt{12}$	0,3					rect/100
Eccentricity $u(I_{ecc,ind}) = 6{,}9\text{x}10^{-5}\ I_j$ Background: G3.5.2	0,4	0,8	1,2	1,7	2,1	rect/100
Creep/hysteresis $u(I_{time}) = 7{,}7\text{x}10^{-5} I_j$ Background: G3.5.3	0	0,92	1,39	1,85	2,31	rect/100
Uncertainty of indication $u(I)$	3,34	3,54	3,80	4,14	4,54	

Quantity or Influence	Load, indication, error in kg Standard uncertainty in kg, or as relative value	distribution / degrees of freedom
Test loads		
Standard weights[16] m_{c1}	6 000	
$u(\delta m_c) = mpe/\sqrt{3}$	0,173	rect/100
$u(\delta m_D) = mpe/\sqrt{3}$	0,173	rect/100
$u(\delta m_B) = 7{,}2 \text{x} 10^{-6}\ m_{c1}$ [17]	0,043	rect/100
$u(m_{c1})$	0,25	triang/>100

Substitution loads L_{subj} ≈	0	6 000	12 000	18 000	24 000	
$L_{Tj} = m_{c1} + L_{subj}$ ≈	6 000	12 000	18 000	24 000	30 000	
$u(\delta m_B) = 2{,}9x10^{-6} L_{sub}$ see G3.5.4	Negligible					
$u(L_{Tj}) = \sqrt{\left\{ j^2 u^2(m_{c1}) + 2\sum u^2(I_{j-1}) \right\}}$	0,25	4,76	6,93	8,80	10,62	triangular to normal / >100
Unc. of error $u(E) = \sqrt{\{u^2(I_j) + u^2(L_{Tj})\}}$	3,36	5,93	7,90	9,74	11,56	
υ_{eff}	4	35	74	100	113	
k (95,45 %)	2,87	2,07	2,03	2,03	2,02	
$U(E) = ku(E)$ / kg	**9,6**	**12,3**	**16,0**	**19,8**	**23,4**	
additional, optional:						
Result of approximation by straight line through zero / kg	$E_{appr}(R) = 0{,}00019R$					
Uncertainty to approximated errors / kg	$u(E_{appr}(R)) = \sqrt{\left(7{,}3 \times 10^{-7}\,\text{kg}^2 + 1{,}75 \times 10^{-7} R^2\right)}$ [18]					
Expanded uncertainty, with k = 2 / kg	$U(E_{appr}(R)) = 2u(E_{appr}(R)) = 8{,}4 \times 10^{-4} R$					

The certificate should give the advice to the user that any reading R obtained after the calibration, should be corrected by subtracting the relevant error E mentioned above only after rounding to the scale interval *d*, symbol E_d, and that the standard uncertainty to the error of any reading is increased by the standard uncertainty of the reading $u(R) = \sqrt{\left(2d^2/12 + s^2\right)} = 5{,}25$ kg.

[16] Class M1, calibrated 3 months ago, average drift monitored over 2 recalibrations $|D_{mc}| \le mpe$ over 12 months; used at nominal value; fairly accomodated to ambient temperature, ΔT < 5 K

[17] Value from Table E2.1 for cast iron, grey: $\rho = (7\ 100 \pm 300)$ kg/m³, $u(\rho_a)$=0,064 kg/m³, $\hat{w}(m_B) = 7{,}2 \text{x} 10^{-6}$ (7.1.2-5a)

[18] The first term is negligible

For actual values that may be presented in the certificate see G3.5.5.

It would be acceptable to state in the certificate only the largest value of expanded uncertainty for all the reported errors: $U(E)=$ **23,4 kg, o** $U(E_d)=$ **25 kg** based on ***k* = 2,02**, accompanied by the statement that the coverage probability is at least 95 %.

G3.4 Uncertainty of indications in use

As stated in 7.4, the following information may be developed by the calibration laboratory or by the user of the instrument. In any case, it may not be presented nor considered as part of the calibration certificate.

G3.4.1 The normal conditions of use of the instrument, as assumed, or as specified by the user may include

Variation of temperature from – 10 °C to + 30 °C
Loads not always centered carefully
Tare balancing function operated
Loading times: normal, that is shorter than at calibration

G3.4.2 Calculation table as per 7.4 and 7.5

Quantity or Influence	Indication in kg Standard uncertainty, relative or in kg	Distribution / degrees of freedom
Errors determined by calibration	$E(R) = 0{,}00019R$	
Standard uncertainty $u(E(R))$	$u(E(R)) = \sqrt{\{(5{,}25\text{kg})^2 + 1{,}75 \times 10^{-7} R^2\}}$	
Further contributions to the uncertainty		
Weighing instrument		
Adjustm't drift: change of $E(Max)$ over 1 year = 15 kg	$\hat{w} = (R_{adj}) = 15/(30000\sqrt{3}) = 2{,}89 \times 10^{-4}$	rect/100
Temperature: $\hat{w}(R_{temp}) = TCx\,\Delta T/\sqrt{12}$	$2 \times 10^{-6} \times 40/\sqrt{12} = 0{,}23 \times 10^{-4}$	rect/100
Weighing procedure		
Eccentricity of load: $\hat{w}(R_{ecc}) = \lvert \Delta I \rvert_{\max} / (L_{ecc}\sqrt{3})$	$5/(10470\sqrt{3}) = 2{,}76 \times 10^{-4}$	rect/100
Tare balancing: non-linearity of errors smaller than their standard uncertainty!	-----	rect/100
Loading time: $u(I_{time})$ applies for the whole weighing range, see G3.5.3	$\hat{w}(R_{time}) = 0{,}77 \times 10^{-4}$	rect/100
Uncertainty of weighing result $u(W)$	$u(W) = \sqrt{(5{,}24\text{ kg})^2 + \begin{pmatrix} 17{,}48 + 8{,}35 + 0{,}053 \\ + 7{,}62 + 0{,}59) \times 10^{-8} R^2 \end{pmatrix}}$ $u(W) = \sqrt{(5{,}25\text{ kg})^2 + 3{,}41 \times 10^{-7} R^2}$	
k (≈95%)	2	
Uncertainty of weighing result with correction by E_{appr}		
$U(W) = ku(W)$	$U(W) = 2\sqrt{(5{,}25\text{ kg})^2 + 3{,}41\text{x}10^{-7} R^2}$	
simplified to first order	$U(W) \approx U(W = 0) + \left\{ \begin{bmatrix} U(W = Max) \\ -U(W = 0) \end{bmatrix} / Max \right\} R$ $U(W) \approx 10{,}5\text{ kg} + 8{,}69 \times 10^{-4} R$	
Global uncertainty of weighing result without correction to the reading		
$U_{gl}(W) = U(W) + \lvert E_{appr}(R) \rvert$	$U_{gl}(W) = 10{,}5\text{ kg} + 1{,}06 \times 10^{-3} R$	

G3.4.3 An attachment to the certificate could contain this statement:

"Under normal conditions of use, including

ambient temperature varying between–10 °C and + 30 °C,
loads applied without special care to apply centre of gravity in centre of load receptor,
obtaining readings *R* with or without tare balancing (Net or Gross values)
not applying any correction to the readings *R*,

the weighing result *W* is

$$W = R \pm \left(10{,}5\,\text{kg} + 1{,}06 \times 10^{-4} R\right)$$

at a level of confidence f better than 95%"

An alternative would read:
(Conditions as before)..".,
the weighing result *W* is

within a tolerance of	1 %	for	$R \geq$ 1 200 kg,
within a tolerance of	0,5 %	for	$R \geq$ 2 800 kg,
within a tolerance of	0,2%	for	$R \geq$ 13 930 kg,

at a level of confidence of better than 95%."

G3.5 Further information to the example

G3.5.1 Details of substitution procedure; reference: 4.3.3

For the calibration tests with load substitution, each substitution load was adjusted by adding or subtracting machinery parts so as to achieve differences of $\Delta I_j \leq 20$ kg (saves time compared with adjustment to ΔI ≤ 1 kg). All indications in high resolution d_T = 1 kg.

At step 1, the empty trailer was used as substitution load; at steps 2 to 4, 2 steel containers were put on the trailer each time.

All data that have been recorded are presented in full hereafter. Consistent with 4.3.3, the symbols are

L_{Tj} test load at step *j*, made up of m_{c1} =6 000 kg standard weights plus the accumulated substitution load L_{Tj-1}

$E_j = I_j - L_{Tj}$

I'_j indication alter removal of m_{c1}

$I(L_{subj})$ indication alter adding ≈ 6 000 kg substitution load

$\Delta I_j = I(L_{subj}) - I_j$

$L_{subj} = L_{Tj} + \Delta I_j$, value of the substitution load

Step j	L_{Tj} kg	I_j kg	E_j kg	I'_j kg	$I(L_{subj})$ kg	ΔI_j kg	L_{subj} kg
0	0	0	0				
1	6 000	6 001	1	1	6 015	14	6 014
2	12 014	12 014	0	6 016	11 996	-18	11 996
3	17 996	17 999	3	12 001	18 017	18	18 014
4	24 014	24 019	5	18 022	24 006	-13	24 001
5	30 001	30 010	9	---		---	

After removal of all test loads, a stable indication of 4 kg was recorded.

In G3.3, all indications are quoted as the nominal values, as per 6.2.1.

G3.5.2 Eccentricity of test loads

Positions of load for eccentricity test: distances from centre of load receptor were

2,50 m in length and 0,75 m in width, as for standard load positions for this test.
Loads for tests of indication were centered carefully by eyesight, a largest distance of 1 m in length and 0,4 m in width was observed. The eccentricity of these loads was therefore not greater than 1/2 of the distances at eccentricity test.
The relative standard uncertainty for eccentricity at indication tests is then

$$\hat{w}\left(I_{ecc,ind}\right) = \left|\Delta I_{ecc}\right|_{\max} / \left(2L_{ecc}\sqrt{12}\right)$$

G3.5.3 Effects of creep and hysteresis

Considered due to the fact that the procedure includes a loading and unloading sequence, and that some extra time is needed for the adjustment of each accumulated substitution load.
For creep and hysteresis, a contribution can be derived from the indication E_0 on return to zero load, as per 7.4.4.2.
Expression (7.4.4-7): $\hat{w}\left(I_{time}\right) = E_0 / \left(Max\sqrt{3}\right)$
Gives a value of

$$\hat{w}\left(I_{time}\right) = 4 / \left(30000\sqrt{3}\right) = 7{,}7\text{x}10^{-5}$$

which should be added to the uncertainty of the indication for all loads except the first 6 000 kg load which consists only of the standard weights

The same uncertainty is added to the indications in use, because the loading time in normal use is expected to be quite short, and therefore differs from that at calibration.

G3.5.4 Air buoyancy correction for substitution loads

The substitution loads were a trailer and steel containers filled with waste machinery parts (steel or cast iron).
For the density of the filled containers, $\rho =$(7 500 ± 400) kg/m³ is assumed (base don the information given in Appendix E).
For the trailer, the same density may be assumed for simplicity's sake (made mostly of steel, except for the tyres and some parts of the brake system).
Over the duration of the calibration, the air temperature t varied from 17 °C to 20 °C, and the atmospheric pressure was $p =$(1 010 ± 10) hPa.
Applying the expression (A1.1-1) in which we neglect the term with relative humidity, we find the extreme values

$$\rho_{a,\min} = 0{,}34848\,p_{\min} / \left(273{,}15 + t_{\max}\right) = 1{,}188\ 9 \text{ kg/m}^3$$
$$\rho_{a,\max} = 0{,}34848\,p_{\max} / \left(273{,}15 + t_{\min}\right) = 1{,}225\ 1 \text{ kg/m}^3$$

With a difference $\Delta\rho_a =$0,036 2 kg/m³
A maximum change of the air buoyancy of the substitution loads would therefore be

$$\Delta m_{sub,B} \approx L_{sub}\Delta\rho_a / \rho = 24\ 000 \times 0{,}036\ 2 / 7\ 500 = 0{,}12 \text{ kg}$$

yielding a relative uncertainty of

$$\hat{w}\left(\delta m_{sub,B}\right) = \Delta m_{sub,B} / \left(L_{sub}\sqrt{3}\right) = 2{,}9 \text{ x } 10^{-6}$$

Which in negligible indeed.

G3.5.5 Weighing results under the conditions of the calibration

The weighing results under the conditions of the calibration $W^* = R - E$ obtained after the calibration when determined for the test points, are as follows

Reading R / kg	**6 000**	**12 000**	**18 000**	**24 000**	**30 000**	
Errors rounded to d / kg	**0**	**0**	**0**	**10**	**10**	
$u(R)$ / kg	5,25					
$u(W^*) = \sqrt{u^2(R) + u^2(E)}$ / kg	6,23	7,92	9,49	11,06	12,70	
υ_{eff}	38	91	133	153	157	
$k\,(95{,}45\%)$	2,07	2,03	2,02	2,02	2,02	
$\boldsymbol{U(W^*) = ku(W^*)}$ / kg	**12,9**	**16,1**	**19,2**	**22,3**	**25,6**	
Result of approximation by straight line through zero / kg	$E_{appr}(R) = 0{,}00019R$					
Uncertainty to W^* / kg	$u(W^*) = 10{,}5\text{ kg} + 5{,}29 \times 10^{-4} R$					
Expanded uncertainty, with k = 2 / kg	$U(W^*) = 2u(W^*)$					